This key is written at the end of the semester. Solutions reflect techniques you know at the end of the semester and not just during Test 1.

MAT 161 Test 1 Name Key

Summer 2013

Instructions: To receive credit for all answers, show all work clearly in the space provided.

1. Find the indicated limits.

a. $\lim_{x\to 1} \dfrac{5x^2-13x-28}{x^2-x-12}$

$= \dfrac{5-13-28}{1-1-12}$

$= \dfrac{-36}{-12} = \boxed{3}$

b. $\lim_{x\to 4} \dfrac{5x^2-13x-28}{x^2-x-12}$ $= \lim_{x\to 4} \dfrac{10x-13}{2x-1}$

L'Hospital

form $\frac{0}{0}$

$= \dfrac{40-13}{2(4)-1}$

$= \boxed{\dfrac{27}{7}}$

or $\lim_{x\to 4} \dfrac{(x-4)(5x+7)}{(x-4)(x+3)} = \dfrac{27}{7}$

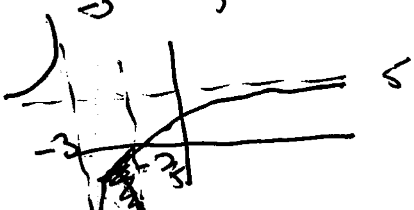
c. $\lim_{x\to -3^+} \dfrac{5x^2-13x-28}{x^2-x-12} = \lim_{x\to -3^+} \dfrac{5x+7}{x+3}$

$x+3$: $-$ $+$ $+$

$5x+7$: $-$ $-$ $+$

$+$ -3 $-$ $-\frac{7}{5}$ $+$

$\lim_{x\to -3^+} \dfrac{5x+7}{x+3} = \boxed{-\infty}$



d. $\lim_{x\to +\infty} \dfrac{5x^2-13x-28}{x^2-x-12}$

$= \boxed{\dfrac{5}{1} \text{ leading coefficients}}$

2. Suppose that for all real numbers x, $-4x^3-6x^2 \le f(x) \le x^4 + 4x + 1$

a. Is that f(x) is continuous at x= -1? Why or why not?

$\lim_{x\to -1} -4x^3-6x^2 = 4-6 = -2 \le f(-1) \le -2 = \lim_{x\to -1} x^4+4x+1 = 2-4$

$\therefore f(-1) = 2 = \lim_{x\to -1} f(x)$

$\therefore$ f is cont. at $x=-1$

b. Is. f(x) continous at x = 1? Why or why not?

There is insufficent evidence to determine that f is cont. at $x=1$

All we know is that

$\lim_{x\to 1^-} f(x) = -10 \le f(1) \le \lim_{x\to 1^+} f(x) = 1+4+1 = 6.$

3. Using the precise (δ, ϵ) definition of limits, prove that $\lim_{x \to 2} 7x - 5 = 9$

Given $\epsilon > 0$ choose $\delta = \frac{\epsilon}{7}$

Then if $0 < |x-2| < \delta$

$|x-2| < \frac{\epsilon}{7}$

$|7x-14| < \epsilon$

$|(7x-5)-9| < \epsilon$.

4. Use the Intermediate Value Theorem to find an interval that contains a solution to the equation

$x^4 = 20x + 100$ in the interval.

$f(x) = x^4 - 20x - 100$ is cont. and $f(0) = -100$

and $f(10) = 10000 - 200 - 100 = 9,70$

0 is between -100 & $9,700$

$\therefore$ there is a z between 0 & 10 $\ni$ $f(z) = 0$

or $z^4 - 20z - 100 = 0$ or $z^2 = 20z + 100$.

5. Let $f(x) = 5x^2 + x$. Find the equation of the tangent line to the curve y = f(x) at (1, 6).

Using the definition of the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5(x+h)^2 + (x+h) - (5x^2 + x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 + \cancel{x} + h - \cancel{5x^2} - \cancel{x}}{h}$$

$$= \lim_{h \to 0} 10x + 5h + 1 = 10x + 1$$

or

$f'(x) = 5(2x) + 1$

$= 10x + 1$

$f'(1) = 10(1) + 1 = 11$

$\boxed{y - 6 = 11(x-1)}$ or $y - 6 = 11x - 11$ or $y = 11x - 5$

6. On a planet far in another galaxy, a stone is thrown vertically upward with an initial velocity of 10 ft/s from a bridge 75 feet above a river. By Newton's Laws of Motion, the position of the stone measured as the height above the river after t seconds is $s(t) = -t^2 + 10t + 75$ where $s = 0$ is the level of the river.

   a. Find $s'(t)$.

   $s'(t) = -2t + 10$

   b. Find the average velocity of the stone from $t = 2$ to $t = 2.1$ seconds.

   $\frac{s(2.1) - s(2)}{.1} = \frac{-(2.1)^2 + 10(2.1) + \cancel{75} - (-2^2 + 10(2) + \cancel{75})}{.1}$

   $= \frac{-((2.1)^2 - 2^2) + 10(.1)}{.1} - \frac{(.1)(4.1) + 10(.1)}{.1} = -4.1 + 10$

   $= 5.9$ ft/sec

   c. Find the instantaneous velocity when $t = 2$ seconds.

   $s'(2) = -2(2) + 10 = 6$ sec

   d. Find the instantaneous velocity when the stone hits the river.

   $s(t) = -t^2 + 10t + 75 = 0$

   $-(t^2 - 10t - 75) = 0$

   $-(t - 15)(t + 5) = 0$

   $t = 15$ hits the river

   $s'(15) = -2(15) + 10$

   $= -20$ ft/sec.

#5 S.B. cube
#6 donotsimplify answers

MAT 161 **Test 2** **Name** Key
Summer 2013
Instructions: To receive credit for all answers, show all work clearly in the space provided.

1. Let $f(x) = 6x^{0.5} - 5\sin(x-1) + 20x$

   a. Find $f'(x)$.

   $f'(x) = 3x^{-\frac{1}{2}} - 5\cos(x-1) + 20$

   10

   b. Find the equation of the tangent line to the curve $y = f(x)$ at (1, 26).

   $f'(1) = 3 - 5 + 20 = 18$ 2

   $y - 26 = 18(x-1)$

   $y = 18x + 8$

2. Find the second derivative of $f(x) = \sin(5 + \ln(2x))$.

   $f'(x) = \cos(5+\ln(2x))\left(0 + \frac{2}{2x}\right) = \frac{\cos(5+\ln(2x))}{x}$

   10 $f''(x) = \frac{x\left(-\sin(5+\ln(2x))\frac{2}{2x}\right) - \cos(5+\ln(2x))(1)}{x^2}$

   $= \frac{-\sin(5+\ln(2x)) - \cos(5+\ln(2x))}{x^2}$

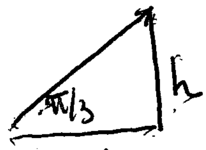
3. Calculate the linearization of $f(x) = x^{\frac{1}{4}}$ at a = 81 Then estimate $(80.95)^{\frac{1}{4}}$ using the linearization. $= 2.99953493$

   2 $\boxed{f(x) = x^{\frac{1}{4}} \quad f(81) = 81^{\frac{1}{4}} = 3}$ $\boxed{f'(x) = \frac{1}{4}x^{-3/4} \quad f'(81) = \frac{1}{4}3^{-3} = \frac{1}{4(27)} = \frac{1}{108}}$ 2

   10 3 $L(x) = 3 + \frac{1}{108}(x-81)$

   $\boxed{3\ L(80.95) = 3 + \frac{1}{108}(-.05) = 3 - \frac{.05}{108}}$ ≈ 2.99953703

4. A jet is climbing at a 60 degree angle to the horizontal. The sun is directly overhead. How fast is the jet gaining altitude if the speed of its shadow on the ground is 400 miles per hour?



1 1

$\frac{dh}{dt} = ?$ $\frac{dx}{dt} = 400$

$\frac{x}{3}$

10

$\boxed{\frac{h}{x} = \tan\frac{\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}}$ 2

1 $h = \sqrt{3}\, x$

$\frac{dh}{dt} = \sqrt{3}\,\frac{dx}{dt} = \sqrt{3}\,(400)$ miles per hour

1 1

692.82

5. The edge of a ~~square~~ cube is measured at 1000 ± 3 cm. Calculate the volume with an estimate for the error using differentials..

10

3 $V = x^3$

$dV = 3x^2\,dx = 3(1000)^2(3) = 9\,10^6$ cm^3

1 1 1 2

$V = 1000^3 = 10^9$

2

In problems 6 – 10, calculate the derivative of y with respect to x.

6. $y = \dfrac{3x + 4^{7x}}{\cos(x^2)}$

$\frac{d}{dx} a^x = a^x \ln a \qquad \frac{d}{dx} a^{f(x)} = a^{f(x)} (\ln a) f'(x)$

$$\frac{dy}{dx} = \frac{(\cos(x^2))(3 + 4^{7x}(\ln 4)(7)) - (3x + 4^{7x})(-\sin(x^2))(2x)}{(\cos(x^2))^2}$$

1 2 1 1 2 1 2 1

7. $y = \ln(\sin x) + (1 + \cos x)^2$

$$\frac{dy}{dx} = 2\frac{\cos x}{\sin x} + 2(1 + \cos x)^{2-1}(-\sin x)$$

2 1 1 1 1 1

8. $y = \cos(\arctan x)$

$$\frac{dy}{dx} = -\sin(\arctan x)\frac{1}{1+x^2}$$

4 3 3

9. $7x^2 - xy^2 + \cos(8y^3) = 7\sin(x) - 4y$

$$14x - 2y^2 - x2y\frac{dy}{dx} - \sin(8y^3)\left(24y^2\frac{dy}{dx}\right) = 7\cos x - 4\frac{dy}{dx}$$

1 1 1 1 1 1

$$2\left(4 - 2xy - 24y^2\sin(8y^3)\right)\frac{dy}{dx} = 7\cos x - 14x + y^2$$

$$\frac{dy}{dx} = \frac{7\cos x - 14x + y^2}{\cancel{14x}\ 2\ 4 - 2xy - 24y^2\sin(y^3)}$$

2

10. $y = (3 + x)^{\ln(2x+4)}$

$$\ln y = \ln(2x+4)\ln(3+x) \quad 2$$

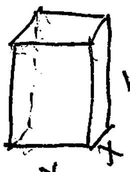
$$\frac{dy}{dx} = y\frac{d\ln y}{dx} = (3+x)^{\ln(2x+4)}\left\{\frac{2}{2x+4}\ln(3+x) + \ln(2x+4)\frac{1}{3+x}\right\}$$

1 1 2 1 1 2

MAT 161
Summer I 2013 **Test 3** **Name** Key

Instructions: To receive credit for all answers, show all work clearly in the space provided.

(15) 1. Find the dimensions of a 500,000 cubic foot steel holding tank, square-based, open-top, that is of minimal surface area.

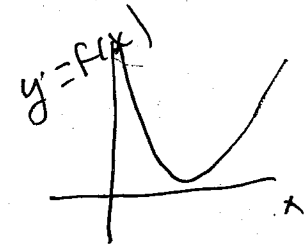


2 volume $= x^2h = 500{,}000 \Rightarrow h = 500{,}000x^{-2}$

Min. Surface $= x^2 + 4xh = x^2 + 4x(500000x^{-2})$

2 min $f(x) = x^2 + 2000000x^{-1}$ on $(0, +\infty)$

5
$f'(x) = 2x - 2000000x^{-2} = 0$
$2x = \frac{2000000}{x^2}$
$x^3 = 1{,}000{,}000$
$x = 100$
$f'(100) = 0$

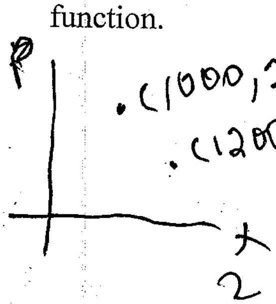


3
$f''(x) = 2 + 4{,}000{,}000x^{-3}$
$f''(100) = 2 + \frac{4 \cdot 10^6}{(100)^3} > 0$
f has a single critical value at $x = 100$ which is a rel. min

$x = 100$

1 $h = \frac{500{,}000}{(100)^2} = 50$

(15) 2. The Seahawk can sell 1000 cdogs per day at a price of \$2.00 per cdog and 1200 cdogs per day at a price of \$1.50 per cdog. To produce and sell x cdogs, the cost to the Seahawk is x+50 dollars per day. What price per cdog should the Seahawk charge in order to maximize profits? Assume a linear demand function.



$m = \frac{2 - 1.5}{1000 - 1200} = \frac{.5}{-200} = \frac{-1}{400}$

2 $p - 2 = -\frac{1}{400}(x - 1000)$

$p = -\frac{1}{400}x + 2.5 + 2$

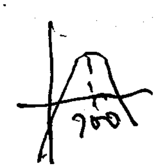
$p = \frac{-1}{400}x + 4.5$

2 Max Profits $= xp - (x + 50)$
$f(x) = -\frac{1}{400}x^2 + 3.5x - 50$

5
$f'(x) = -\frac{1}{200}x + 3.5 = 0$
$x = 3.5(200) = 700$

3
$f''(x) = -\frac{1}{200}$
$f'(700) = 0$; $f''(700) < 0$

f is max when $x = 700$



1 $p = -\frac{700}{400} + 4.5$
$= -1.75 + 4.5$
$= \boxed{\$2.75}$

3. Calculate $\lim_{x \to 0} \frac{x - \sin(x)}{4x^3}$. $= \lim_{x \to 0} \frac{1 - \cos x}{12x^2}$ 3

$= \lim_{x \to 0} \frac{\sin x}{24x}$ 3

10

$= \lim_{x \to 0} \frac{\cos x}{24}$ 3 $= \frac{1}{24}$ 1

4. Find f(x) if f "(x) = 3x² + cos(x) + 1 and f(0) = 2 and f(π) = 1.

$f'(x) = 3\frac{x^3}{3} + \sin(x) + x + k_1$

14

$f(x) = \frac{1}{4}x^4 - \cos(x) + \frac{1}{2}x^2 + k_1 x + k_2$

$2 = f(0) = 0 - 1 + 0 + 0 + k_2$

$3 = k_2$

$f(x) = \frac{1}{4}x^4 - \cos(x) + \frac{1}{2}x^2 + k_1 x + 3$

$1 = f(\pi) = \frac{1}{4}\pi^4 - (-1) + \frac{1}{2}\pi^2 + k_1\pi + 3$

$\frac{-3 - \frac{1}{4}\pi^4 - \frac{1}{2}\pi^2}{\pi} = k_1$

5. Use an initial guess of 10 and Newton's Method once to estimate the solution to $x^2 + x - 109 = 0$.

$x - \frac{f(x)}{f'(x)} = x - \frac{x^2 + x - 109}{2x + 1}$ 3

10 3

$= 10 - \frac{10^2 + 10 - 109}{2(10) + 1}$ 4 $= 10 - \frac{1}{21}$

36

6. For $f(x) = x^5 - 5x$

a. Calculate the first and second derivative of $f(x)$.

6

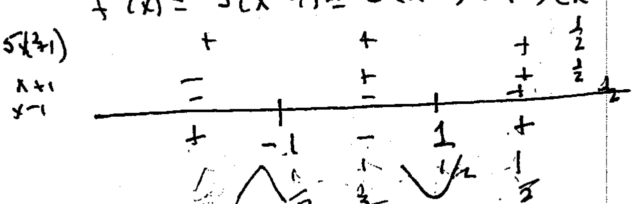
$f'(x) = 5x^4 - 5$ 2

$f''(x) = 20x^3$ 2

b. Find the intervals where $f(x)$ is increasing and decreasing.

6

$f'(x) = 5(x^4-1) = 5(x-1)(x+1)(x^2+1)$



1 f is inc on $(-\infty,-1) \cup (1,+\infty)$

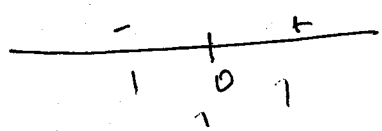
1 f is dec on $(-1,1)$

2

c. Find the intervals where $f(x)$ is concave up and concave down.

6

$f''(x) = 20x^3$ 1



1 f is concave down on $(-\infty, 0)$

1 f is concave ~~down~~ up on $(0, +\infty)$

d. Identify the local maximum, local minimum, and inflection points.

6

2 f has a rel max at $x=-1$

2 f " " " min at $x=1$

2 f has an inflection point at $x=0$

e. Find the x- and y-intercepts of $y = f(x)$.

6

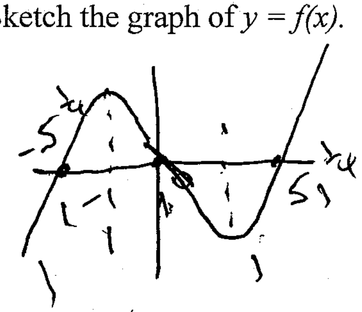
$f(x) = x^5 - 5x = x(x^4-5) = x(x^2-\sqrt{5})(x^2+\sqrt{5}) = x(x-\sqrt{\sqrt{5}})(x+\sqrt{\sqrt{5}})(x^2+\sqrt{5})$

$f(0)=0$ $(0,0)$ is the y-intercept 3

1 $x=0$, $x=5^{\frac{1}{4}}$, $x=-5^{\frac{1}{4}}$ are the x-intercepts 1 1

f. Sketch the graph of $y = f(x)$.

6



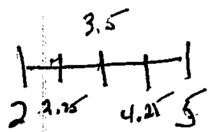
MAT 161 Test 4 Name Key

Summer I, 2013

Show all work for credit purposes.

1. Evaluate the Riemann sum for $f(x) = 3x^2 + 5$ on $2 \le x \le 5$, with four subintervals, taking the sample points to be the right endpoints.

(16)



$\Delta x = \frac{5-2}{4} = \frac{3}{4}$

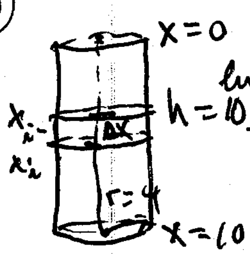
$\frac{3}{4}\left(f(2.75) + f(3.5) + f(4.25) + f(5)\right)$

$= \frac{3}{4}(208.625) = \frac{5007}{32}$

$= 156.46875$

2. An upright cylindrical tank of radius 4 feet and height 10 feet is filled with water weighing 62.5 lbs per cubic foot. How much work is required to pump out the water through the top of the tank.

(10)



$\lim_{n\to\infty} \sum_{i=1}^{n} x_i \cdot 62.5\pi 4^2 \Delta x = \int_0^{10} x\left(62.5\pi(4^2)\right)dx$

$= \frac{x^2}{2}(62.5)\pi(16)\Big|_0^{10}$

$= 50(62.5)\pi(16)$ ft-lb.

$= 157079.6327$ ft-lb.

3. Calculate $\lim_{n\to\infty} \sum_{k=1}^{n} \left(3 + k\frac{4}{n}\right)^8 \left(\frac{4}{n}\right)$ by evaluating the equivalent integral.

(10)

$b - a = 4$

$a = 3$

$b = 7$

$f(x) = x^8$

$\int_3^7 x^8\,dx = \frac{1}{9}x^9\Big|_3^7$

$= \frac{1}{9}(7^9 - 3^9)$

$= 4481547.\overline{1}$

4. Find the area from $x = 2$ to $x = 5$, between the x-axis and the curve $y = 3x^2 - \frac{1}{x}$.

(10)

$$\int_2^5 \left(3x^2 - \frac{1}{x}\right)dx = \left(x^3 - \ln x\right)\Big|_2^5$$

$$= (125 - \ln 5) - (2^3 - \ln 2)$$

$$= 117 + \ln 2 - \ln 5 = 117 + \ln .4$$

$$= 116.0837$$

5. Find the average value of $f(x) = x + e^x$ on the interval [1, 11].

(10)

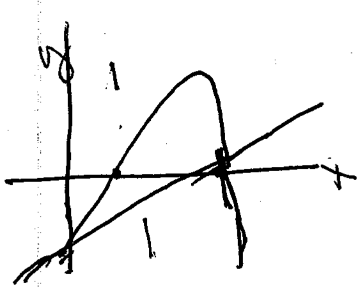
$$\frac{1}{11-1}\int_1^{11} (x + e^x)\,dx = \frac{1}{10}\left(\frac{1}{2}x^2 + e^x\right)\Big|_1^{11}$$

$$= \frac{1}{10}\left(\frac{1}{2}(11)^2 + e^{11} - \left(\frac{1}{2} + e\right)\right)$$

$$= 5993.14$$

6. Calculate the area bounded by the curves $y = (x-2)(1-x)$ and $y = x - 2$.

(10)



$\frac{10}{16}$

end 3/13

$$x - 2 = (x-2)(1-x)$$

$$0 = (x-2)(1-x) - (x-2)$$

$$0 = (x-2)[1 - x - 1]$$

$$= (x-2)(-x)$$

$$x = 0,\ x = 2$$

$$\int_0^2 [(x-2)(1-x) - (x-2)]\,dx = \int_0^2 (-x^2 + 2x)\,dx$$

$$= -\frac{1}{3}x^3 + x^2 \Big|_0^2$$

$$= -\frac{8}{3} + 4 = \boxed{\frac{4}{3}}$$

7. Calculate the following.

20

a. $\int\left(\frac{3}{\sqrt{1-x^2}}+\sin x\right)dx$ 16

$\boxed{3\arcsin x - \cos x + C}$

2 2 2

b. $\int x^3\cos(x^4+1)dx$ 7

1 $w = x^4+1$

1 $\frac{dw}{dx} = 4x^3$

1 $\frac{1}{4}dw = x^3dx$

1 $\int \cos w \frac{1}{4}dw$

1 $\frac{1}{4}\sin w + C$

$\boxed{\frac{1}{4}\sin(x^4+1)+C}$

1 1

c. $\int_{-1}^{2}(5x+5)^4\,dx$ 7

1 $w = 5x+5$

1 $\frac{dw}{dx} = 5$

1 $\frac{1}{5}dw = dx$

1 $\int_0^{15} w^4 \frac{1}{5}dw$

1 $\frac{1}{25}w^5 \Big|_0^{15}$

2 $\frac{1}{25}(15)^5$

30375

8. Find the derivative of the following function of x: $\int_{-1}^{2x}\cos(5t+5)^4\,dt$

10

$y = \int_{-1}^{u}\cos(5t+5)^4\,dt$ 4 1 1 $u = 2x$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \cos(5u+5)^4 \quad 2 \quad 3$

1 1 $= \boxed{\cos(10x+5)^4\,(2)}$ 2

9. A particle is moving in a straight line with a velocity of 2t+3 feet per second from t= 3 to t=5 seconds. How far has the particle moved during that 2 second interval?

10

$S(5)-S(3) = \int_3^5 v(t)\,dt = \int_3^5(2t+3)\,dt$ 2

2 2

$=(t^2+3t)\Big|_3^5$ 2

$=(5^2+3\cdot5) - (3^2+3)$ 2

= 128

40−12= 22