Prerequisites

Preview – velocity $s(t) = 256 - 16t^2$
– area $f(t) = 256 - 16t^2$

2.1
Tangent + Velocity Problems

1.1 Four ways to represent a function
(a) verbally description in words
(b) numerically by a table of values
(c) visually by a graph
(d) algebraically by an explicit formula

def.
vertical line test
piecewise – defined
increasing / decreasing

1.2 Mathematical Models
linear
polynomials
power functions
rational
algebraic
trigonometric functions
exponential functions
logarithmic functions

1.3 vertical and horizontal shifts

$c > 0$	$f(x+c)$	$c > 1$	$cf(x)$
	$f(x-c)$		$\frac{1}{c} f(x)$
	$f(x) + c$		$f(cx)$
	$f(x) - c$		$f(\frac{x}{c})$
			$-f(x)$
			$f(-x)$

$+, -, *, /, \circ$ five ways to combine function

1.4 Graphing Calculators + Computers

1.5 exponential $f(x) = a^x$ / 1.6 inverse + log functions
horizontal line test

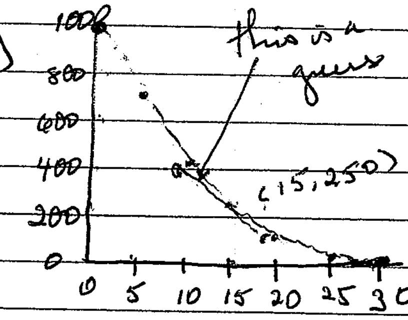
①

t	v
0	1000
5	694
10	444
15	250
20	111
25	28
30	0

(a) slope between (5, 694) & (15, 250) -44.4
" " (10, 444) & " " -38.8
" " (20, 111) & (15, 250) -27.8
" " (25, 28) & (15, 250) -22.2
" " (30, 0) & (15, 250) $-16.\overline{6}$

(b) average of -38.8 & -27.8 $= -33.3$
or average of -44.4 & -22.2 $= -33.3$
Answer is -33.3

(c)



tangent line passes through (10, 400) and (19, 100) approximately

$m = \frac{400-100}{10-19} = \frac{300}{-9} \approx -33.\overline{3}$

Page 86 problem ends here

Later we will see that $V(t) = 1000\left(1-\frac{t}{30}\right)^2$

$$\frac{V(t+h)-V(t)}{t+h-t} = \frac{1000\left(1-\frac{t+h}{30}\right)^2 - 1000\left(1-\frac{t}{30}\right)^2}{h}$$

Newton Quotient from $(t, V(t))$ to $(t+h, V(t+h))$

$$= \frac{1000\left[\left(1-\frac{t+h}{30}\right)^2 - \left(1-\frac{t}{30}\right)^2\right]}{h}$$

$$= \frac{1000\left[\left(2-\frac{t+h}{30}-\frac{t}{30}\right)\left(\left(1-\frac{t+h}{30}\right)-\left(1-\frac{t}{30}\right)\right)\right]}{h}$$

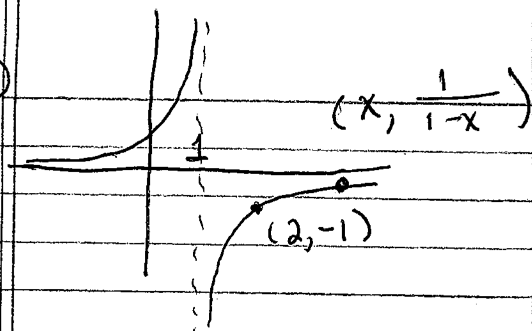
$$= \frac{1000\left[2-\frac{t+h}{30}-\frac{t}{30}\right]\left[-\frac{h}{30}\right]}{h}$$

$$= -\frac{1000}{30}\left[2-\frac{2t}{30}-\frac{h}{30}\right] \rightarrow \frac{-100}{3}\left[2-\frac{t}{15}\right] \text{ as } h \to 0.$$

$-\frac{100}{3}\left(2-\frac{t}{15}\right)$ is the slope of tangent line at $(t, V(t))$

at $t = 15$, this is $-\frac{100}{3} = -33.3$

③



The slope of the secant line from $(x, \frac{1}{1-x})$ to $(2,-1)$ is

$$\frac{\frac{1}{1-x} - -1}{\cancel{2-x}\; x-2} = \frac{\frac{1}{1-x}+1}{\cancel{2-x}\; x-2} = \frac{\frac{1}{1-x}+\frac{1-x}{1-x}}{\cancel{2-x}\; x-2} = \frac{2-x}{\underset{x-2}{\cancel{2-x}})(1-x)} = \frac{-1}{1-x}$$

$$= \frac{1}{x-1} = \text{slope}$$

(a)

x	slope
1.5	$\frac{1}{.5} = 2$
1.9	$\frac{1}{.9} = 1.\overline{1}$
1.99	$\frac{1}{.99} = 1.\overline{01}$
1.999	$\frac{1}{.999} = 1.\overline{001}$
2.5	$\frac{1}{1.5} = .\overline{6}$
2.1	$\frac{1}{1.1} = .\overline{90}$
2.01	$\frac{1}{1.01} = .\overline{9900}$
2.001	$\frac{1}{1.001} = .\overline{999000}$

(b) slope $= 1$

(c) $y-(-1) = 1(x-2)$

$y+1 = x-2$

$\boxed{y = x-3}$

$\boxed{\text{Note } \frac{1}{x-1} \to 1 \text{ as } x \to 2.}$

⑤ $y = 40t - 16t^2$ Note the height when $t=2$ is

$y = 40(2) - 16(2^2) = 80 - 64 = 16$ feet.

Average Velocity from $t=2$ seconds to $2+h$ seconds is

$$\frac{\text{distance}}{\text{time}} = \frac{40(2+h) - 16(2+h)^2 - 16}{2+h-2} = \frac{\cancel{80}+40h - 16(\cancel{4}+4h+h^2) - \cancel{16}}{h}$$

$$= 40 - 16(4+h) = -24 - 16h$$

h	Av. Velocity
.5	$-24-16(.5) = -32$ ft/sec
.1	$-24 - 1.6 = -25.6$ ft/sec
.05	$-24 - 16(.05) = -24.8$ ft/sec
.01	$-24 - 16(.01) = -24.16$ ft/sec

(b) instant. velocity is -24

(Note $-24-16h \to -24$ as $h \to 0$.

2.2. The limit of a function
2.3 Calculating Limits Using the Limit Laws
2.4 The Precise Definition of a limit

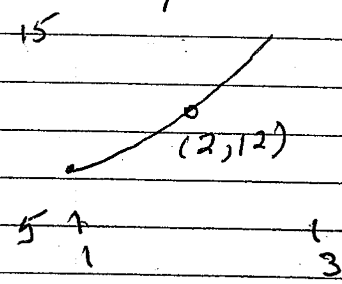
2.2 Example $f(x) = \dfrac{x^3-8}{x-2}$ Note $f(x)$ is defined in a neighborhood of 2, but not necessarily at 2 itself.

What is happening near 2?

Numerical Approach

x	$f(x)$
1.9	11.41
1.99	11.94
1.999	11.994
2.001	12.006
2.01	12.06
2.1	12.61

Graphical



Algebraic

$f(x) = \dfrac{x^3-8}{x-2} = \dfrac{(x-2)(x^2+2x+4)}{x-2}$
$= x^2+2x+4$
except at 2
But $2^2+2(2)+4 = 12$

We say $\lim_{x\to 2} f(x) = 12$ meaning that
① $f(x)$ is defined near 2
② as x gets arbitrarily close to 2, then $f(x)$ gets arbitrarily close to 12.
This will be made more precise in 2.4.

$\therefore \lim_{x\to a} f(x) = L$ means that $f(x)\to L$ as $x \to a$.

Ex 2 Page 89 Estimate $\lim_{t\to 0} \dfrac{\sqrt{t^2+9}-3}{t^2}$

Numerical

t	$f(t)$
± 1	.16228
$\pm .5$	.16553
$\pm .1$	.16662
$\pm .05$	.16666
$\pm .01$	.16667
$\pm .001$	.1666667

$\lim_{t\to 0} f(t) = \frac{1}{6}$

Graphical
see figure 5

Use TableSet & Table
① start -10^{-5} inc 10^{-6}
② start -10^{-6} inc 10^{-7}

Algebraic

$\dfrac{\sqrt{t^2+9}-3}{t^2} \cdot \dfrac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}$

$\dfrac{t^2}{t^2(\sqrt{t^2+9}+3)}$

$\dfrac{1}{\sqrt{t^2+9}+3}$

at $t=0$, $\dfrac{1}{\sqrt{9}+3} = \dfrac{1}{6}$

Concepts: $\lim_{x \to a} f(x) = L$ $\quad \lim_{x \to a^+} f(x) = L$ $\quad \lim_{x \to a^-} f(x) = L$

Ex: $\lim_{x \to 2^+} \frac{|x-2|}{x-2} = 1$

$\lim_{x \to 2^-} \frac{|x-2|}{x-2} = -1$

$\lim_{x \to 2} \frac{|x-2|}{x-2}$ does not exist

Note! $\lim_{x \to a} f(x) = L$ iff $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$

Concept: $\lim_{x \to a} f(x) = +\infty$ $\quad \lim_{x \to a} f(x) = -\infty$

$\lim_{x \to a^+} f(x) = +\infty$ $\quad \lim_{x \to a^+} f(x) = -\infty$

$\lim_{x \to a^-} f(x) = +\infty$ $\quad \lim_{x \to a^-} f(x) = -\infty$

Vertical Asymptotes

In class #20   HW #19
#26   #23
#30   #29
#32   #31

2.3 Suppose $\lim_{x \to a} f(x) = L_1$ $\quad \lim_{x \to a} g(x) = L_2$

Then $\lim_{x \to a} (f(x) \pm g(x)) = L_1 \pm L_2$

$\lim_{x \to a} (f(x) g(x)) = L_1 L_2$

if $L_2 \neq 0$ $\quad \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L_1}{L_2}$

$\lim_{x \to a} c f(x) = c L_1$

n positive integer $\quad \lim_{x \to a} (f(x))^n = L_1^n$

$\lim_{x \to a} c = c$ $\quad \lim_{x \to a} x = a$

$\lim_{x \to a} x^n = a^n$ $\quad n$ positive integer

$\lim_{x \to a} x^{\frac{1}{n}} = a^{\frac{1}{n}}$

$\lim_{x \to a} (f(x))^{\frac{1}{n}} = L_1^{\frac{1}{n}}$ if definable

f polynomial or rational $\quad a$ in domain of f

$\lim_{x \to a} f(x) = f(a)$.

Squeeze Law ① $f(x) \leq h(x) \leq g(x)$ in some punctured neighborhood of c ② $\lim_{x \to c} f(x) = L = \lim_{x \to c} g(x)$

Then $\lim_{x \to c} h(x) = L$.