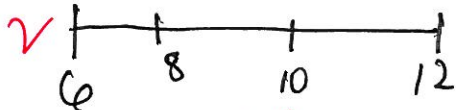
MAT 161 Test 4 Name Key
Spring 2016

Show all work for credit purposes.

1. Evaluate the Riemann sum for $f(x) = 200x - x^2$ on $6 \le x \le 12$, with three subintervals, taking the sample points to be the midpoints.

10



$\Delta x = \frac{12-6}{3} = 2$ ✓

$f(7)2 + f(9)2 + f(11)(2)$ 3

$(1351)(2) + (1719)(2) + 2079(2)$ 3

$(5149)(2)$

10298

2. Calculate $\lim_{n\to\infty} \sum_{k=1}^{n} \left(1 + k\frac{4}{n}\right)^3 \left(\frac{4}{n}\right)$ by evaluating the equivalent integral.

10

$b - a = 4$ 1
$a = 1$ 1
$b = 5$ 1

$\int_1^5 x^3\,dx = \frac{1}{4}x^4\Big|_1^5 = \frac{1}{4}(5^4 - 1^4) = \frac{1}{4}(624) = 156$
1 1 2 2

3. Find the area from $x = 2$ to $x = 4$, between the x-axis and the curve $y = 2x + \sin(\pi x)$.

10

$\int_2^4 (2x + \sin(\pi x))\,dx = \left(x^2 - \frac{\cos(\pi x)}{\pi}\right)\Big|_2^4 = 4^2 - 2^2 = 12$
1 1 1 2 2 2

4. Find the average value of $f(x) = 6x + 5e^x$ on the interval [1, 3].

10

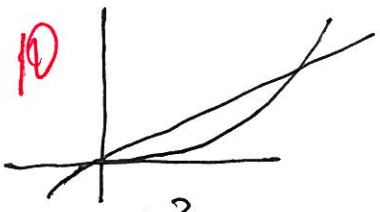
$\frac{1}{3-1}\int_1^3 (6x + 5e^x)\,dx = \frac{1}{2}(3x^2 + 5e^x)\Big|_1^3$
1 1 1 2 1

$= \frac{1}{2}(27 + 5e^3 - (3 + 5e))$ 2 55.41813774
1 1

/40

5. Calculate the area bounded by the curves $y = 0.1x^2$ and y = x.

10



$.1x^2 = x$

$.1x^2 - x = 0$

$x(.1x - 1) = 0$

$x = 0$ & $x = 10$

$$\int_0^{10} (x - .1x^2)\,dx = \left(\frac{1}{2}x^2 - \frac{.1}{3}x^3\right)\Big|_0^{10}$$

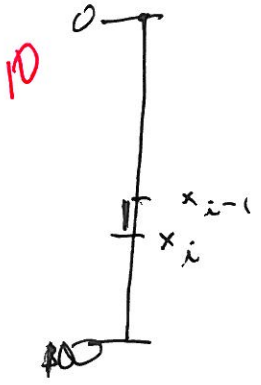
$$= \frac{1}{2}(10^2) - \frac{.1}{3}10^3$$

$$= \frac{1}{2}10^2 - \frac{1}{3}10^2$$

$$= \frac{1}{6}(10^2)$$

6. A 300-lb. cable is 100 ft. long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?

10



weight $= 3\Delta x$

height $= x_i$

work $= 3x_i\,dx$

$$\int_0^{100} 3x\,dx = \frac{3}{2}x^2\Big|_0^{100}$$

$$= \frac{3}{2}(10000)$$

$$= 15000 \text{ ft-lbs}$$

7. Find the derivative of $F(x) = \int_{-1}^{4x} \sin(\cos(6t) + 5)\,dt$

10

$$y = \int_{-1}^{u} \sin(\cos(6t) + 5)\,dt \qquad u = 4x$$

$$\frac{dy}{dt} = \frac{dy}{du}\frac{du}{dx} = \sin(\cos(6u) + 5)\,4$$

$$= \sin(\cos(24x) + 5)\,4$$

/30

8. Calculate the following.

10

a. $\int\left(\frac{x}{36+x^2}\right)dx$

$u = 36+x^2$ 2

$\frac{du}{dx} = 2x$ 1

$\frac{1}{2}du = x\,dx$ 1

$\int \frac{1}{u}\,\frac{1}{2}du$ 2

$\frac{1}{2}\ln u + C$ 2 2

$\boxed{\frac{1}{2}\ln(36+x^2)+C}$

10

b. $\int \cos(x)\ \sec^2(\sin(x)+3)dx$

$u = \sin(x)+3$ 2

$du = \cos x\,dx$ 2

$\int \sec^2 u\,du$ 2

$\tan u + C$ 2 2

$\boxed{\tan(\sin(x)+3)}$ +0

10

c. $\int_0^2 x^2(3x^3+7)^4\,dx$

$u = 3x^3+7$ 2

$du = 9x^2dx$ 1

$\frac{1}{9}du = x^2dx$ 1

$\int u^4\,\frac{1}{9}du$ 1

$\frac{1}{45}u^5 + C$ 1

$\boxed{\frac{1}{45}(3x^3+7)^5+C}$ 1

$\frac{1}{45}[(36)^5 - 7^5]$

~~$\frac{5250\,5008}{45}$~~ $\frac{28612344}{45}$

635829.8667