MAT 161 Test 4 Name Key

Spring 2016 Make-Up

Show all work for credit purposes.

1. Evaluate the Riemann sum for $f(x)=7+x^2$ on $-1\le x\le 5$, with three subintervals, taking the sample points to be the midpoints.

-1 $\quad$ 5

$\Delta x = \frac{5--1}{3} = \frac{6}{3} = 2$

$x_i = -1 + i2$

$x_0 = -1 \quad x_2 = 3$

$x_1 = 1 \quad x_3 = 5$

$\Delta x(f(0)+f(2)+f(4))$

$= 2((7+0^2)+(7+2^2)+(7+4^2))$

$= 2(21+4+16)$

$= 2(41)$

$= 82$

2. Calculate $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+k\frac{7}{n}\right)^4\left(\frac{7}{n}\right)$ by evaluating the equivalent integral.

$\Delta x = \frac{b-a}{n} = \frac{7}{n} \Rightarrow b-a=7$

$x_k = a + k\Delta x = 3+k\frac{7}{n} \Rightarrow a=3 \Rightarrow b=10$

$f(x)=x^4$

$\int_3^{10} x^4\,dx = \frac{1}{5}x^5\Big|_3^{10} = \frac{1}{5}(10^5-3^5) = 19951.4$

3. Find the area from $x=0$ to $x=4$, between the x-axis and the curve $y=2x^3-\cos(\pi x)$.

$\int_0^4 (2x^3-\cos\pi x)\,dx = \frac{2}{4}x^4 - \frac{\sin \pi x}{\pi}\Big|_0^4$

$= \frac{1}{2}4^4 - \frac{\sin 4\pi}{\pi} - (0-0)$

$= \frac{1}{2}4^4 = 128$

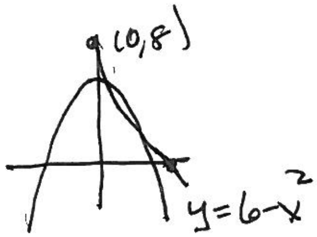
4. Find the average value of $f(x)=2x+4e^{3x}$ on the interval [0, 2].

$\frac{1}{2-0}\int_0^2 (2x+4e^{3x})\,dx = \frac{1}{2}\left(x^2+\frac{4}{3}e^{3x}\right)\Big|_0^2$

$= \frac{1}{2}\left(2^2+\frac{4}{3}e^6-\left(0^2+\frac{4}{3}e^0\right)\right)$

$= \frac{1}{2}\left(4-\frac{4}{3}+\frac{4}{3}e^6\right) = 203.71$

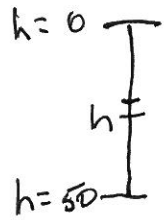
5. Calculate the area bounded by the curves $y = 6 - x^2$ and y = -3x+8.



$$6 - x^2 = -3x + 8$$
$$0 = x^2 - 3x + 2$$
$$= (x-2)(x-1)$$
$$x = 2 \text{ and } x = 1$$

$$\int_1^2 [(6-x^2) - (-3x+8)]\,dx$$
$$\int_1^2 (-2 - x^2 + 3x)\,dx$$
$$= \left(-2x - \tfrac{1}{3}x^3 + \tfrac{3}{2}x^2\right)\Big|_1^2$$
$$= -4 - \tfrac{8}{3} + 6 - \left(-2 - \tfrac{1}{3} + \tfrac{3}{2}\right)$$
$$= \boxed{\tfrac{1}{6}}$$

6. A 500-lb. cable is 50 ft. long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?



Δh

$10\Delta h$

$h\,10\,\Delta h$

$$\int_0^{50} 10h\,dh$$
$$5h^2\Big|_0^{50} = 5(50^2) \text{ ft-lbs} = \boxed{12,500 \text{ ft-lbs.}}$$

7. Find the derivative of $F(x) = \int_{-1}^{2x} \sin(\cos(6t)+5)^t\,dt$

$$y = \int_{-1}^{u} \sin(\cos(6t)+5)^t\,dt \qquad u = 2x$$
$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \sin(\cos(6u)+5)^u(2)$$
$$= \boxed{2\sin(\cos(12x)+5)^{2x}}$$

8. Calculate the following.

a. $\int\left(\frac{x^3}{36+x^4}\right)dx$

$u = 36 + x^4$

$\frac{du}{dx} = 4x^3$

$\frac{1}{4}du = x^3 dx$

$\int \frac{1}{u}\frac{1}{4}du$

$\frac{1}{4}\ln u + C$

$\boxed{\frac{1}{4}\ln(36+x^4)+C}$

b. $\int e^x \sec^2(e^x+3)dx$

$u = e^x + 3$

$\frac{du}{dx} = e^x$

$du = e^x dx$

$\int \sec^2(u)\,du$

$\tan u + C$

$\boxed{\tan(e^x+3)+C}$

c. $\int_0^2 x^2(2x^3+7)^4 dx$

$u = 2x^3 + 7$

$\frac{du}{dx} = 6x^2$

$\frac{1}{6}du = x^2 dx$

$\int_7^{23} u^4 \frac{1}{6}du$

$\frac{1}{6}\frac{u^5}{5}\Big|_7^{23}$

$\frac{1}{30}\left[23^5 - 7^5\right]$

$\boxed{213984.5\bar{3}}$