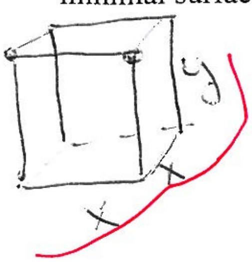
MAT 161 **Test 3** **Name** Key
Spring 2016
Instructions: To receive credit for all answers, show all work clearly in the space provided.

1. Find the dimensions of a 100 cubic foot steel box with an open-top and a square base, that is of minimal surface area.



1

Volume $= 100 = x^2 y \Rightarrow y = 100x^{-2}$ 2

Min. SA $= x^2 + 4xy = x^2 + 4x(100x^{-2})$ 2

$f(x) = x^2 + 400x^{-1}$ 1

$f'(x) = 2x - 400x^{-2} = 0$

$2x = \frac{400}{x^2}$

$x^3 = 200$

$x = (200)^{\frac{1}{3}}$

3

15

$f''(x) = 2 + 800x^{-3}$

$f''(200^{\frac{1}{3}}) > 0$

3

f has a rel min at $x = 200^{\frac{1}{3}}$

and only one critical point on $(0, \infty)$

2

1

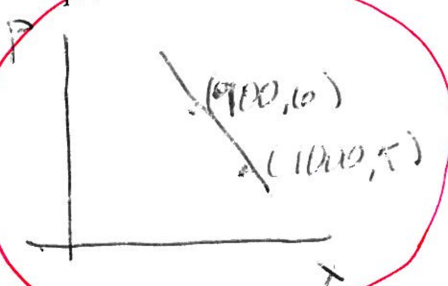
$x = 200^{\frac{1}{3}} = 5.848$

$y = 100(200^{-2/3}) = 2.9240$

f has absolute min

2. Assume a linear demand equation and a total cost function of C(x) = 2x + 100 dollars to produce and sell x widgets per day. 1000 widgets can be sold per day at a price of 5 dollars per widget and 900 widgets per day can be sold at a price of 6 dollars. What should the price be in order to maximize profits ?

15



1

$m = \frac{6-5}{900-1000} = -\frac{1}{100}$

$p - 5 = -\frac{1}{100}(x - 1000)$ 3

$p = -\frac{1}{100}x + 10 + 5$

$p = -\frac{1}{100}x + 15$

2

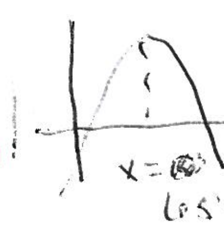
Max $xp - C(x) = -\frac{1}{100}x^2 + 15x - 2x - 100$

$f(x) = -\frac{1}{100}x^2 + 13x - 100$ 1

$\rightarrow f''(x) = -\frac{1}{50}$

$f'(x) = -\frac{1}{50}x + 13 = 0$

$x = 50(13) = 650$ 3



f max when x = 650

$p = -\frac{1}{100}(650) + 15$

$= -6.5 + 15$

$= 8.5$ dollars

30

3. Calculate $\lim_{x\to 0} \dfrac{1+\sin(3x) \ -\cos(5x)}{4x}$.

$$\lim_{x\to 0} \frac{\cos(3x)(3)+\sin(5x)5}{4} = \frac{1(3)}{4} = \frac{3}{4}$$

8 2 2 2 2

4. Find f(x) if f "(x) = 30x , f '(0) = -6 and f(0) = 10 .

$f'(x) = 15x^2 + C_1$ 2

$-6 = f'(0) = C_1$ 2

$f'(x) = 15x^2 - 6$ 1

10 5

$f(x) = 5x^3 - 6x + C_2$ 2

$10 = f(0) = 5(0^3) - 6(0) + C_2$ 2

5 $\boxed{f(x) = 5x^3 - 6x + 10}$ 1

5. Use an initial guess of 1 and Newton's Method once to estimate the solution to $x^3 - 5x+3 = 0$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_0 - \frac{x_0^3 - 5x + 3}{3x_0^2 - 5}$$

2 2

8

$$= 1 - \frac{1-5+3}{3-5} = 1 - \frac{-1}{-2} = \frac{1}{2}$$

2 2

6. How far does it take a car to ~~sop~~ stop if it is traveling 60 miles per hour when a constant braking deceleration of -4ft per second squared is applied?

$a = -4$ 1 $V_0 = \frac{60 \text{ mi}}{1 \text{ hour}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ hour}}{3600 \text{ sec}} = 60\left(\frac{5280}{3600}\right)$ ft/sec

8

2 $s(t) = -2t^2 + \frac{60(5280)}{3600} t + 0$

2 $v(t) = -4t + \frac{60(5280)}{3600} = 0$

2 $t = \frac{60(5280)}{4(3600)}$? Answer

$\to s\left(\frac{60}{4}\frac{(5280)}{3600}\right)$

1 $= -2\left(\frac{60}{4}\left(\frac{5280}{3600}\right)\right)^2 + \frac{60^2(5280)^2}{4(3600)^2}$

$= \frac{60^2(5280)^2}{8(3600)^2}$ $= \frac{(5280)(5280)}{\dots}$ ≈ 968 feet

7. For $f(x) = x^3(x-5)$ $= x^4 - 5x^3$

a. Calculate the first and second derivative of $f(x)$.

(6) (3) $f'(x) = 3x^2(x-5) + x^3 = x^2(3x - 15 + x) = x^2(4x-15)$

(3) $f''(x) = 2x(4x-15) + x^2(4)$

$= 2x(4x-15) + 2x) = 2x(6x-15)$

b. Find the intervals where $f(x)$ is increasing and decreasing.

(6)

x^2	+	+	+
$4x-15$	−	−	+
	−	−	+

$0 \qquad \frac{15}{4}$

2

f is inc on $(\frac{15}{4}, +\infty)$ 2

f is dec on $(-\infty, -\frac{15}{4})$ 2

c. Find the intervals where $f(x)$ is concave up and concave down.

(6)

$2x$	−	+	+
$6x-15$	−	−	+
	(+)	(−)	(+)

$0 \qquad \frac{15}{6}$

2

2 f is concave up on $(-\infty, 0) \cup (\frac{15}{6}, +\infty)$

2 f is concave down on $(0, \frac{15}{6})$

d. Identify the local maximum, local minimum, and inflection points.

(6) local min at $x = \frac{15}{4}$ 2

no local max 2

inflection points at 0 + $\frac{15}{6}$ 2

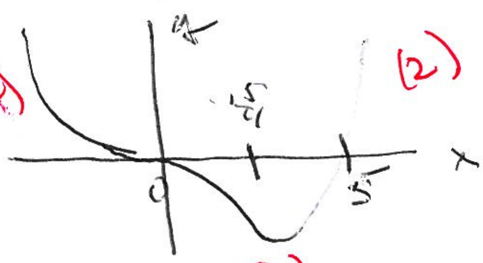
e. Find the x- and y-intercepts of $y = f(x)$.

(6) $f(0) = 0$ $(0,0)$ y-intercept 3

$(0,0)$ + $(5,0)$ x-intercepts 3

f. Sketch the graph of $y = f(x)$.

(6) (2) (2)



(2)

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