MAT 161 Test 2 Name Key
Spring 2016

For complete credit, show all work.

1. A population grows at a constant relative growth rate; at t= 1 hour, the population is 5 and at t = 5 hours the population is 10. Find the relative growth rate and determine the population when t = 9 hours.

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2 $P(t) = Ce^{kt}$

1 $5 = P(1) = Ce^{k}$

1 $10 = P(5) = Ce^{5k}$

1 $2 = e^{4k}$

2 $\frac{1}{4}\ln 2 = k \approx .173287$

1 $5 = Ce^{\frac{1}{4}\ln 2} = C\,2^{\frac{1}{4}}$

1 $C = 5(2^{-\frac{1}{4}}) \approx 4.204448$

$P(t) = 5(2^{-\frac{1}{4}})\,e^{(\frac{1}{4}\ln 2)t}$ 1

$P(9) = 5(2^{-\frac{1}{4}})\,e^{(\frac{1}{4}\ln 2)9}$ 1

$= 5(2^{-\frac{1}{4}})\,2^{9/4}$ 1

$= 5\,2^{8/4} = 5(2^2) = \boxed{20}$ 1

2. Calculate the linearization of $f(x) = x^{\frac{5}{4}}$ at $a = 16$ Then estimate $(16.02)^{\frac{5}{4}}$ — that is $(16.02)^{5/4}$ using the linearization.

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$f(x) = x^{5/4}$ 2 $f(16) = 16^{5/4} = 2^5 = 32$ 2

$f'(x) = \frac{5}{4}x^{\frac{1}{4}}$ 2 $f'(16) = \frac{5}{4}(16^{\frac{1}{4}}) = \frac{5}{2}$ 2

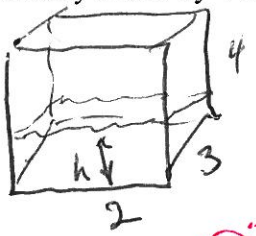
$y - 32 = \frac{5}{2}(x-16)$ or $y = 32 + \frac{5}{2}(x-16)$ 3

at $x = 16.02$ $y = 32 + \frac{5}{2}(.02) = 32 + 5(.01) = \boxed{32.05}$

1 1

3. Water is leaking at the rate of ½ cubic feet per hour from an aquarium in the shape of a rectangular box 2 feet by 3 feet by 4 feet high. At what rate is water level falling when the water level is 1 foot?

13



2

the

2 $V = 6h$

2 $\frac{dV}{dt} = 6\frac{dh}{dt}$

2 $-\frac{1}{2} = 6\frac{dh}{dt}$

2 $\frac{dh}{dt} = -\frac{1}{12}$ foot/hour

$\frac{dV}{dt} = -\frac{1}{2}$ $\frac{dh}{dt} = ?$ when $h = 1$ (3)

In problems ~~8-10~~ 4-9, calculate the derivative of y with respect to x.

4. $f(x) = x^3\sqrt{1+2x^5} + 5x + 4.$

10

$$\frac{dy}{dx} = 3x^2(1+2x^5)^{\frac{1}{2}} + x^3\,\tfrac{1}{2}(1+2x^5)^{-\frac{1}{2}}(10x^4) + 5 + 0$$

2 1 1 2 2 2

5. $y = \dfrac{\log(2x)}{9^{8x}}$

10

$$\frac{dy}{dx} = \frac{9^{8x}\left(\frac{1}{2x\ln 10}\cdot 2\right) - \log(2x)\left(9^{8x}\ln 9\cdot 8\right)}{(9^{8x})^2}$$

2 2 1 2 2 1

6. $y = \sin(\cos(x)) + \arctan(x)$

10

$$\frac{dy}{dx} = \cos(\cos x)(-\sin x) + \frac{1}{1+x^2}$$

3 2 5

7. $\ln(y^2 - 5xy) + 3xy = (7x + y)$

10

$$\frac{1}{y^2-5xy}\left(2y\frac{dy}{dx} - 5y - 5x\frac{dy}{dx}\right) + 3y + 3x\frac{dy}{dx} = 7 + \frac{dy}{dx}$$

$$\frac{dy}{dx} = 7 + \frac{5y}{y^2-5xy} - 3y$$

8. $y = \arcsin(5x) - \cos(x^2)$

10

$$\left(\frac{1}{y^2-5xy}\right)(2y - 5x) + 3x - 1$$

$$\frac{dy}{dx} = 3\,\frac{1}{\sqrt{1-(5x)^2}}^{\frac{2}{5}} + \sin(x^2)(2x)$$

3 2

9. $y = (5x^2 + 3)^{\sin(x)}$

10

$$\ln y = \sin x \ln(5x^2+3)$$

$$\frac{dy}{dx} = (5x^2+3)^{\sin x}\left\{\cos x \ln(5x^2+3) + \sin x \frac{10x}{5x^2+3}\right\}$$

1 1 2 1 1 1 2