

**Instructions:** To receive credit for all answers, show all work clearly in the space provided. You may use graphing calculators. This is designed to be a 50 minute test.

1. Find the indicated limits. If the limit does not exist, tell why.

a.  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{2x^2 - 10x}$

b.  $\lim_{x \rightarrow 6} \frac{x^2 - 25}{2x^2 - 10x}$

c.  $\lim_{x \rightarrow 0} \frac{x^2 - 25}{2x^2 - 10x}$

d.  $\lim_{x \rightarrow -\infty} \frac{x^2 - 25}{2x^2 - 10x}$

2. a. Suppose that for all real numbers  $x$ ,  $70\sqrt{x} \leq f(x) \leq 25x + 49$ , Is  $f(x)$  continuous at  $49/25$  ? Why or why not?

b. Is the function  $f(x)$  in question 2a continuous at  $x = 2$ ? Why or why not?

3. Using the precise  $(\delta, \varepsilon)$  definition of limits, prove that  $\lim_{x \rightarrow 2} (-3x + 4) = -2$

4. Suppose  $f(x)$  and its derivative both have all real numbers as a domain and the graph of  $y = f(x)$  has a horizontal asymptote of  $y = 4$ .

a. What is  $\lim_{x \rightarrow +\infty} f(x)$  ?

b. What is  $\lim_{x \rightarrow +\infty} \frac{f(x+h) - f(x)}{h}$  if  $h$  is a fixed nonzero number?

5. Use the Intermediate Value Theorem to find an interval where there is a solution to the equation  $8.1 = 6x - x^2$  in the interval.

6. a. Use the definition of a derivative to find  $f'(x)$  where  $f(x) = 5x^2 + 3x$ .

b. Find the equation of the tangent line to the graph of  $y = f(x)$  at  $(2, 26)$ ?

c. Find the instantaneous rate of change of  $y = f(x)$  with respect to  $x$  when  $x = -2$ .

d. Find the average rate of change of  $y = f(x)$  with respect to  $x$  over the interval  $[-1, 1]$ ?

7. Find  $f'(3)$  if  $f(x) = \frac{1+2x}{2+x}$ .