MAT 161 **Test 1** **Name** Key

Spring 2016

Instructions: To receive credit for all answers, show all work clearly in the space provided. You may use graphing calculators. This is designed to be a 50 minute test.

1. Find the indicated limits. If the limit does not exist, tell why.

a. $\lim_{x \to 5} \dfrac{x^2 - 25}{2x^2 - 10x}$

5

$\lim_{x \to 5} \dfrac{(x-\cancel{5})(x+5)}{2x(\cancel{x-5})}$ 3

2 $\dfrac{10}{2(5)} = \dfrac{10}{10} = \boxed{1}$

b. $\lim_{x \to 6} \dfrac{x^2 - 25}{2x^2 - 10x}$ $= \dfrac{6^2 - 25}{2(6)^2 - 60}$ 2 2

5

$= \dfrac{11}{72-60} = \boxed{\dfrac{11}{12}}$ 1

c. $\lim_{x \to 0} \dfrac{x^2 - 25}{2x^2 - 10x}$ does not exist 2

5

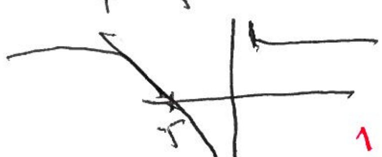
x+5 − + +

x − − +

+ −5 − 0 +

$\lim_{x \to 0^+} f(x) = +\infty$ 1

$\lim_{x \to 0^-} f(x) = -\infty$ 1



1

d. $\lim_{x \to -\infty} \dfrac{x^2 - 25}{2x^2 - 10x}$ $= \dfrac{1}{2}$ leading coefficients 2

5

3

2. a. Suppose that for all real numbers x, $70\sqrt{x} \le f(x) \le 25x + 49$, Is f(x) continuous at 49/25 ? Why or why not? 4

10

4

$g(x) = 70\sqrt{x}$

$g\left(\frac{49}{25}\right) = 70\left(\frac{7}{5}\right) = 14(7) = 98$

$h(x) = 25x + 49 = 25\left(\frac{49}{25}\right) = 49 + 49 = 98$

$98 \le f\left(\frac{49}{25}\right) \le 98$

$\therefore f\left(\frac{49}{25}\right) = 98$

$\lim_{x \to \frac{49}{25}} 70\sqrt{x} = 70\left(\sqrt{\frac{49}{25}}\right) = 98$

$\lim_{x \to \frac{49}{25}} 25x + 49 = 25\left(\frac{49}{25}\right) + 49 = 98$

$\lim_{x \to \frac{49}{25}} f(x) = 98$

2

$\therefore \lim_{x \to \frac{49}{25}} f(x) = f\left(\frac{49}{25}\right)$ $\therefore$ f is cont at $\frac{49}{25}$

b. Is the function f(x) in question 2a continuous at x = 2? Why or why not?

5 2 $g(2) = 70\sqrt{2} \le f(x) \le 25(2) + 49 = 99$

2 $\because 70\sqrt{2} \ne 99 \Rightarrow$ we cannot determine f(2)

1 $\therefore$ There is insufficient evidence to conclude f is cont. at 2.

3. Using the precise (δ, ε) definition of limits, prove that $\lim_{x \to 2} (-3x+4) = -2$

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Given $\epsilon > 0$ choose $\delta = \frac{\epsilon}{3}$ 2

$\forall$ $0 < |x - 2| < \delta$ 2

$|x - 2| < \epsilon/3$ 1

$|3x - 6| < \epsilon$ 1

$|-3x + 6| < \epsilon$ 1

$\Rightarrow$ $|-3x + 4 + 6 - 4| < \epsilon$

$|(-3x+4) + 2| < \epsilon$ 1

$|(-3x+4) - (-2)| < \epsilon$. 1

4. Suppose f(x) and its derivative both have all real numbers as a domain and the graph of y = f(x) has a horizontal asymptote of y = 4.

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a. What is $\lim_{x \to +\infty} f(x)$?

4 Because of the horizontal asymptote of y = 4

3 2

b. What is $\lim_{x \to +\infty} \frac{f(x+h) - f(x)}{h}$ if h is a fixed nonzero number?

$\lim_{x \to +\infty} \frac{f(x+h) - f(x)}{h} = \frac{4-4}{h} = \frac{0}{h} = 0$

3 1 1

5. Use the Intermediate Value Theorem to find an interval where there is a solution to the equation $8.1 = 6x - x^2$ in the interval.

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2 $f(x) = 6x - x^2 - 8.1$

2 $f(0) = -8.1$

2 $f(3) = 6(3) - 3^2 - 8.1$

$= 18 - 9 - 8.1$

$= 9 - 8.1 = .9$

2 0 is between -8.1 & $.9$

2 $\therefore \exists\, z \in (0, 3)$ so that $f(z) = 0$.

6. a. Use the definition of a derivative to find $f'(x)$ where $f(x) = 5x^2 + 3x$.

$$f'(x) = \lim_{h\to 0} \frac{5(x+h)^2 + 3(x+h) - (5x^2+3x)}{h} \quad 2$$

$$= \lim_{h\to 0} \frac{5(\cancel{x^2} + 2xh + h^2) + \cancel{3x} + 3h - \cancel{5x^2} - \cancel{3x}}{h} \quad 2$$

7

$$= \lim_{h\to 0} 5(2x+h) + 3 \quad = 5(2x) + 3 \quad 1$$

$$= 10x + 3$$

2

b. Firnd the equation of the tangent line to the graph of y = f(x) at (2, 26) ?

$f'(2) = 10(2) + 3 = 23$ 3

6

$\boxed{y - 26 = 23(x-2)}$

1 1 1

c. Find the instantaneous rate of change of y = f(x) with respect to x when x = -2.

4

$f'(-2) = 10(-2) + 3 = -20+3 = -17$

3 3

d. Find the average rate of change of y = f(x) with respect to x over the interval [-1 , 1] ?

$$2 \quad \frac{f(1) - f(-1)}{1 - -1} = \frac{5+3-(5-3)}{2} = \frac{8-2}{2} = 4-1 = \boxed{3} \quad 2$$

6 2

7. Find f"(3) if $f(x) = \dfrac{1+2x}{2+x}$.

$$\lim_{h\to 0} \frac{\frac{1+2(x+h)}{2+x+h} - \frac{1+2x}{2+x}}{h} \quad 1 \quad 1$$

10

$$= \lim_{h\to 0} \frac{(1+2x+2h)(2+x) - (1+2x)(2+x+h)}{h(2+x+h)(2+x)} \quad 2$$

$$= \lim_{h\to 0} \frac{\cancel{(1+2x)(2+x)} + 2h(2+x) - \cancel{(1+2x)(2+x)} - h(1+2x)}{h(2+x+h)(2+x)} \quad 2$$

$$= \lim_{h\to 0} \frac{2(2+x) - (1+2x)}{(2+x+h)(2+x)} \quad 1 = \frac{4+2x-1-2x}{(2+x)^2} \quad 1 = \frac{3}{(2+x)^2}, \quad 0$$

$f'(3) = \dfrac{3}{(2+3)^2} = \dfrac{3}{25}$