MAT 161 Final Exam Name _____
Friday, December 4, 2015

1. Calculate the following limits:

   a. $\lim_{x\to 3} \dfrac{3x^2+5x-28}{x^2-16}$

   b. $\lim_{x\to+\infty} e^{-x}(x+1)^2$

2. Use the definition of the derivative to find $f'(x)$ when $f(x) = x^2 + 6x + 18$.

3. Give a delta-epsilon argument that $\lim_{x\to 2} 8x - 5 = 11.$

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4. Tell why $f(x) = |x^2 - 25|$ is continuous at x =5 or not. Is f(x) differentiable there? Why or why not.

5. Calculate $\frac{dy}{dx}$ for the following:

a. $y = \left[x^2 \sin(5x) - 4\right]^{10}$

b. $y = \dfrac{x^3 + 8x}{e^{\frac{x}{2}} + 2}$

c. $y = 4^{5x} + \tan^{-1}(3x)$

d. $y = (\cos(2x))^{(9x+17)}$

e. $y^4 + \arcsin y = (x+3)^3 + 2y$

f. $y = \int_0^2 (t \sin(t^2) + 3)^5 \, dt$

6. Use a linear approximation to estimate $1000.2^{\frac{2}{3}}$

7. Find the equation of the tangent line to the curve $y = 5x^2 + 8x^4$ at (1,13).

8. Two small airplanes depart ILM at 3:00 PM. One flies north at 200 MPH and the other west at 150 MPH. At what rate are the planes separating at 4:00 PM?

9. What are the dimensions of a box of minimum cost of materials if the volume is to be 8000 cubic feet, the base is square, and the top is closed? The vertical sides cost $4.00 per square foot and the top and bottom cost $7.00 per square foot.

10. For $f(x) = x^5 - 60x^3$

a. Calculate the first and second derivative of *f(x).*

b. Find the intervals where *f(x)* is increasing and decreasing.

c. Find the intervals where *f(x)* is concave up and concave down.

d. Identify the local maximum, local minimum, and inflection points.

e. Find the *x*- and *y*-intercepts of $y = f(x)$.

f. Sketch the graph of $y = f(x)$.

11. Calculate

a. $\int_1^6 \left(x^2 + \sin(x)\right) dx$

b. $\int \frac{e^{2x}}{(1+e^{4x})}\, dx$

12. Use Newton's Method to find the next guess for the solution of $x^4 = 25x + 3$ if the first guess is $x_0 = 3$

13. Find the area from $x = -1$ to $x = 3$ between the x-axis and the curve $y = e^{3x} + 2$.

14. Evaluate the Riemann Sum for $f(x) = \frac{10}{x} + 2,\ 4 \le x \le 20$, with four subintervals, taking the sample points to be midpoints.

15. Find $f(x)$ if $f''(x) = x^3 + 5x$ and $f(0) = 2$ and $f'(0) = 4$.

16. Find the average value of $f(x) = x + 6x^5$ on [0,2].

17. *a.* Calculate the area between the curves $y = (3 - x)(x-7)$ and $y = 3$.

b. Rotate the area in (*a*) about the *y*-axis and give the integral that determines the value. Do not evaluate the integral.

c. Rotate the area in (*a*) about the *x*-axis and give the integral that determines the value. Do not evaluate the integral.

18. Bill's Hot Dogs can sell 20,000 hot dogs per day at a price of $1.50 per hot dog and 25,000 hot dogs per day at a price of $1.40 per hot dog. Assume a linear demand curve. It costs Bill exactly $1.10 per hot dog to buy and prepare each hot dog. What price should Bill charge per hot dog in order to maximize profits.

Extra Credit: An upright cylindrical tank is a radius of 6 feet and a height of 25 feet is filled with water (62.5 lbs/cubic foot). How much work is required to pump the water out the top of the tank?