Key Fall 2015 v.1

1a. $\lim_{x\to 3} \frac{3x^2+5x-28}{x^2-16} = \frac{27+15-28}{9-16} = \frac{14}{-7} = \boxed{-2}$

b. $\lim_{x\to+\infty} \frac{(x+1)^2}{e^x} = \lim_{x\to+\infty} \frac{2(x+1)}{e^x} = \lim_{x\to+\infty} \frac{2}{e^x} = \boxed{0}$

2. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+6(x+h)+18-(x^2+6x+18)}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2+\cancel{6x}+6h+\cancel{18}-\cancel{x^2}-\cancel{6x}-\cancel{18}}{h}$

$= \lim_{h\to 0} (2x+h+6) = \boxed{2x+6}$

3. Given $\epsilon>0$ choose $\delta = \epsilon/8$.
then if $0<|x-2|<\delta$, then $|x-2|<\epsilon/8$

$|8x-16|<\epsilon$

$|(8x-5)-11|<\epsilon$.

4. $f(x)=|x^2-25|$ is cont. at $x=5$ since $f(5)=0=\lim_{x\to 5}|x^2-25|$

$f(x)$ is not diff at $x=5$ since $\lim_{x\to 5^-} \frac{|x^2-25|-f(5)}{x-5} = \lim_{x\to 5^-} \frac{|x-5|(x+5)}{x-5} = -10$

and $\lim_{x\to 5^+} \frac{|x^2-25|-f(5)}{x-5} = \lim_{x\to 5^+} \frac{(x+5)(x-5)}{x-5} = 10$

$\therefore$ $f'(5)$ does not exist.

5 (a) $\frac{dy}{dx} = 10(x^2\sin(5x)-4)^9[2x\sin(5x)+x^2\cos(5x)5+0]$

(b) $\frac{dy}{dx} = \frac{(e^{x/2}+2)(3x^2+8x)-(x^3+8x)(e^{x/2}(\frac{1}{2})+0)}{(e^{x/2}+2)^2}$

(c) $\frac{dy}{dx} = 4^{5x}(\ln 4)5 + \frac{1}{1+(3x)^2}(3)$

(d) $\ln y = (9x+17)\ln(\cos(2x))$

$\frac{dy}{dx} = \cos(2x)^{9x+17}\left\{9 + 9x+17\left(\frac{-\sin(2x)2}{\cos(2x)}\right)\right\}$

5(e) $4y^3\frac{dy}{dx} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 3(x+3)^2 + 2\frac{dy}{dx}$

$\frac{dy}{dx}\left(4y^3 + \frac{1}{\sqrt{1-y^2}} - 2\right) = 3(x+3)^2$

$$\boxed{\frac{dy}{dx} = \frac{3(x+3)^2}{4y^3 + \frac{1}{\sqrt{1-y^2}} - 2}}$$

(f) $y = \int_0^2 (t\sin(t^2)+3)^5\,dt$ is a constant,

$$\boxed{\frac{dy}{dt} = 0}$$

(6) $f(x) = x^{2/3}$; $f'(x) = \frac{2}{3}x^{-1/3}$

$f(1000) = 1000^{2/3} = 100$; $f'(x) = \frac{2}{3}\frac{1}{1000^{1/3}} = \frac{2}{300}$

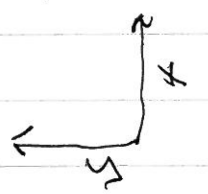
$L(x) = 100 + \frac{2}{300}(x - 1000)$

$L(1000.2) = 100 + \frac{2}{300}(.2) = \boxed{100 + \frac{.4}{300}}$

(7) $\frac{dy}{dt} = 10x + 32x^3$; $\frac{dy}{dx} = 10 + 32 = 42$

$\boxed{y - 13 = 42(x - 1)}$

(8)



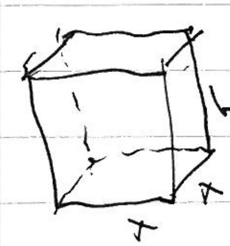
$\frac{dx}{dt} = 200$ $\frac{dy}{dt} = 150$ $\frac{dL}{dt}$ at $t = 1{:}00$

$L^2 = x^2 + y^2$

$2L\frac{dL}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{dL}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{L} = \boxed{\frac{(200)^2 + (150)^2}{\sqrt{(200)^2 + (150)^2}}}$

(9)



~~Mass~~ $x^2h = 8000 \Rightarrow h = 8000x^{-2}$

Min Cost $\underset{C(x)}{} = 7(2x^2) + 4(4xh) = 14x^2 + 16(8000)x^{-1}$

$C'(x) = 28x - 16(8000)x^{-2} = 0$

$28x = \frac{16(8000)}{x^2}$

$x^3 = \frac{16(8000)}{28} = \frac{4(8000)}{7}$

$\to x = \left(\frac{4}{7}\right)^{1/3}(20)$

$C''(x) = 28 + \frac{16(8000)(2)}{x^3}$

$C''\left(\left(\frac{4}{7}\right)^{1/3}(20)\right) > 0$

$\Downarrow$ C is min when $x = \left(\frac{4}{7}\right)^{1/3}(20)$

$f(x) = x^5 - 60x^3 = x^3(x^2 - 60)$

(10) (a) $f'(x) = 5x^4 - 180x^2 = 5x^2(x^2 - 36) = 5x^2(x-6)(x+6)$

$f''(x) = 20x^3 - 360x = 20x(x^2 - 18) = 20x(x - 3\sqrt{2})(x + 3\sqrt{2})$

(b) $x+6$, $5x^2$, $x-6$: $(+)\ -6\ (-)\ 0\ (-)\ 6\ (+)$

f is inc on $(-\infty, -6) \cup (6, +\infty)$

f is dec on $(-6, 6)$

(c) $x+3\sqrt{2}$, $20x$, $x-3\sqrt{2}$: $(-)\ -3\sqrt{2}\ (+)\ 0\ (-)\ 3\sqrt{2}\ (+)$

f is concave up on $(-3\sqrt{2}, 0) \cup (3\sqrt{2}, \infty)$

f is concave down on $(-\infty, -3\sqrt{2}) \cup (0, 3\sqrt{2})$

(d) max at $x = -6$

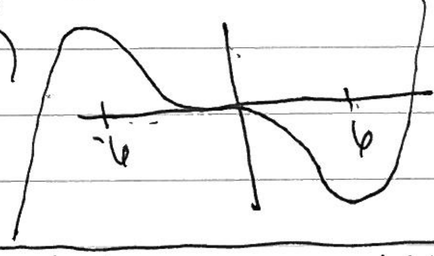
min at $x = 6$

~~on~~ inflection points at $\pm 3\sqrt{2}$ and 0

(e) $(0,0)$ y-intercept

$(\pm\sqrt{60}, 0)$ & $(0,0)$ x-intercepts

(f)



(11) (a) $\left(\frac{1}{3}x^3 - \cos(x)\right)\Big|_1^6 = \frac{1}{3}6^3 - \cos(6) - \left(\frac{1}{3}(1) - \cos(1)\right)$

(b) $u = e^{2x}$

$du = 2e^{2x}$

$\frac{1}{2}du = e^{2x}\,dx$

$\int \frac{e^{2x}}{(1+e^{4x})}\,dx = \int \frac{1}{1+u^2}\,\frac{1}{2}\,du$

$= \frac{1}{2}\arctan u + C$

$= \frac{1}{2}\arctan(e^{2x}) + C$

(12) $x - \frac{f(x)}{f'(x)} = x - \frac{x^4 - 25x + 3}{4x^3 - 25}$

$= 3 - \frac{3^4 - 25(3) + 3}{4(3^3) - 25}$

(13) $\int_{-1}^{3} (e^{3x} + 2)\,dx = \left(\frac{e^{3x}}{3} + 2x\right)\Big|_{-1}^{3} = \frac{e^9}{3} + 2(3) - \left(\frac{e^{-3}}{3} - 2\right)$

(14) $4 \quad 8 \quad 12 \quad 16 \quad 20$ $\quad \frac{b-a}{n} = \frac{20-4}{4} = 4$ $\quad \left[\frac{10}{6} + 2 + \frac{10}{10} + 2 + \frac{10}{14} + 2 + \frac{10}{18} + 2\right] 4$

(15) $f''(x) = x^3 + 5x \qquad f(0) = 2 \qquad f'(0) = 4$

$f'(x) = \frac{1}{4}x^4 + \frac{5}{2}x^2 + c$

$c = f'(0) = 4$

$f'(x) = \frac{1}{4}x^4 + \frac{5}{2}x^2 + \cancel{4}\ 4x$

$f(x) = \frac{1}{20}x^5 + \frac{5}{6}x^3 + \quad + k$

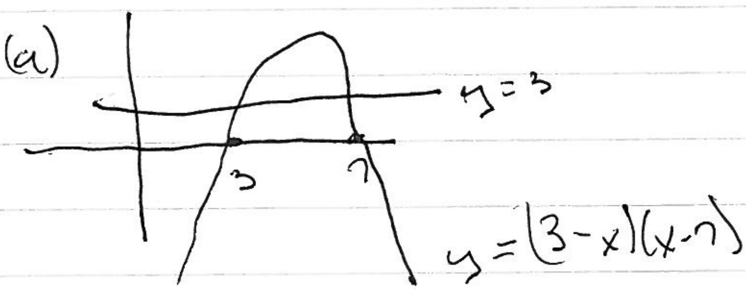
$2 = f(0) = k$

$\boxed{f(x) = \frac{1}{20}x^5 + \frac{5}{6}x^3 + 4x + 2}$

(16) $\frac{1}{2-0}\int_0^2 (x + 6x^5)\,dx = \frac{1}{2}\left(\frac{1}{2}x^2 + x^6\right)\Big|_0^2$

$= \frac{1}{2}\left(\frac{1}{2}2^2 + 2^6\right)$

(17) (a)



$3 = (3-x)(x-7)$

$3 = -21 + 7x + 3x - x^2$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$\int_4^6 \left[(3-x)(x-7) - 3\right]dx = \int_4^6 (-x^2 + 10x - 24)\,dx$

$= \left(-\frac{1}{3}x^3 + 5x^2 - 24x\right)\Big|_4^6$

$= -\frac{1}{3}6^3 + 5(6^2) - 24(6) + \frac{1}{3}4^3 - 5(4^2) + 24(4)$

(b) $\int_4^6 2\pi x(x^2 - 10x + 24)\,dx$

(c) $\int_4^6 \pi\left((3-x)^2(x-7)^2 - 3^2\right)dx$