MAT 161 Test 4 Name Key

Fall 2016 Version 2

Show all work for credit purposes.

1. Evaluate the Riemann sum for $f(x) = 8x + x^3$ on $-2 \le x \le 22$, with four subintervals, taking the sample points to be the midpoints.

(10)

$-2 \qquad 22$

$\Delta x = \frac{b-a}{n} = \frac{22--2}{4} = \frac{24}{4} = 6$

$x_0 = -2$

$x_1 = 4$

$x_2 = 10$

$x_3 = 16$

$x_4 = 22$

$\Delta x(f(1) + f(7) + f(13) + f(19))$

$= 6(8+1 + 8(7)+7^3 + 8(13)+13^3 + 8(19)+19^3)$

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2. Calculate $\lim_{n\to\infty} \sum_{k=1}^{n} \left(5 + k\frac{4}{n}\right)^6 \left(\frac{8}{n}\right)$ by evaluating the equivalent integral.

(10)

$\lim_{n \to +\infty} \frac{8}{4} \sum_{k=1}^{n} \left(5+k\frac{4}{n}\right)^6 \left(\frac{4}{n}\right)$

$\Delta x = \frac{b-a}{n} = \frac{4}{n} \Rightarrow b-a=4$

$a=5$

$b=9$

$f(x) = x^6$

$\int_5^9 x^6\,dx = \frac{1}{7}x^7 \Big|_5^9$

$= \frac{1}{7}(9^7 - 5^7)$

$= 470{,}4844 \approx 472120.5$

3. Find the area from $x = 0$ to $x = \pi$, between the x-axis and the curve $y = 2 + \cos(3x)$.

(10)

$\int_0^{\pi} (2 + \cos(3x))\,dx = \left(2x + \frac{\sin(3x)}{3}\right)\Big|_0^{\pi}$

$= 2\pi + \frac{\sin(3\pi)}{3} - \left(2(0) + \frac{\sin(0)}{3}\right)$

$= 2\pi$

4. Find the average value of $f(x) = 2x + 4x^{-2}$ on the interval [7, 10].

$\frac{1}{10-7} \int_7^{10} (2x + 4x^{-2})\,dx = \frac{1}{3}(x^2 - 4x^{-1})\Big|_7^{10}$

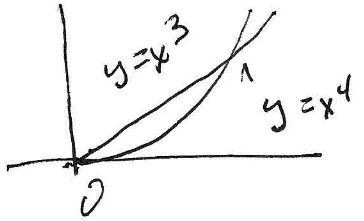
$= \frac{1}{3}\left(10^2 - \frac{4}{10} - \left(7^2 - \frac{4}{7}\right)\right)$

$= \frac{1}{3}\left(51 - 4\left[\frac{1}{10} + \frac{1}{7}\right]\right)$

$= \frac{1}{3}\left(51 - 4\left(\frac{17}{70}\right)\right)$

$= \frac{1}{3}\left(51 - \frac{68}{70}\right) = \frac{1}{3}\left(50 + \frac{1}{35}\right) \approx 16.676$

5. Calculate the area bounded by the curves $y = x^4$ and $y = x^3$

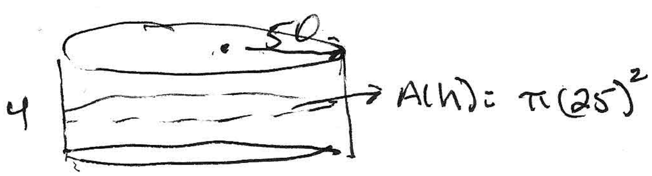


$1 \quad \int_0^1 (x^3 - x^4)\,dx = \left(\frac{1}{4}x^4 - \frac{1}{5}x^5\right)\Big|_0^1 \quad 2$

$= \frac{1}{4} - \frac{1}{5} \quad 2$

$= \frac{1}{20}$

6. A cylinder is 4 ft. tall with a radius of 50 feet is filled with water that weighs 64.5 lbs per cubic foot. How much work is required to empty the cylinder from the top?



1 Volume $= \pi(50^2)\Delta h$

1 weight $= 64.5(\pi\,50^2\,\Delta h)$

1 Work $= 64.5\pi(50^2)\,h\,\Delta h$

1 $\int_0^4 64.5\pi(50^2)\,h\,dh$

1 $64.5\pi(50^2)\frac{h^2}{2}\Big|_0^4$

2 $64.5\pi(50)^2\,8$

405 2654.52 ft-lb.

7. Find the derivative of $F(x) = \int_{10}^{15x} \ln(\ln(6t) + 5)'\,dt$

$y = \int_{10}^{u} \ln(\ln(6t) + 5)\,dt \quad 2 \qquad u = 15x \quad 2$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad 1$

$= \ln(\ln 6u + 5)\,15 \quad 1$

$= \ln(\ln 90x + 5)\,15 \quad 1$

8. Calculate the following.

a. $\int\left(\frac{x^2}{1+x^6}\right)dx$

1 $\int \frac{x^2}{1+(x^3)^2}dx$

2 $u = x^3$

1 $\frac{du}{dx} = 3x^2$

1 $\frac{1}{3}du = x^2 dx$

1 $\int \frac{1}{1+u^2}\frac{1}{3}du$

2 $\frac{1}{3}\arctan u + C$

2 $\frac{1}{3}\arctan x^3 + C$

b. $\int \sin(5x)\sec^2(\cos(5x)\ +3)dx$

2 $u = \cos 5x + 3$

2 $\frac{du}{dx} = -\sin(5x)5$

2 $-\frac{1}{5}du = \sin(5x)\,dx$

1 $\int \sec^2 u \; -\frac{1}{5}du$

1 $-\frac{1}{5}\tan u + C$

2 $-\frac{1}{5}\tan(\cos 5x + 3) + C$

c. $\int_0^2 x^4(2x^5+3)^5\,dx$

$u = 2x^5 + 3$ 2

$\frac{du}{dx} = 10x^4$ 2

$\frac{1}{10}du = x^4 dx$ 2

$\int_3^{67} u^5 \frac{1}{10}du$ 2

$= \frac{1}{10}\frac{u^6}{6}\Big|_3^{67}$ 2

$= \frac{1}{60}(67^6 - 3^6)$

$= 1507{,}639{,}691$