MAT 161 | Test 4 | Name Key
Fall2016 | Version 1

Show all work for credit purposes.

1. Evaluate the Riemann sum for $f(x) = 8x + x^3$ on $-2 \le x \le 14$, with four subintervals, taking the sample points to be the midpoints.

(10)

$$\Delta x = \frac{14 - -2}{4} = \frac{16}{4} = 4$$

$x_0 = -2$, $x_1 = 2$, $x_2 = 6$, $x_3 = 10$, $x_4 = 16$

$$\Delta x\left(f(0) + f(4) + f(8) + f(12)\right)$$

$$4\left(8(0) + 0^3 + 8(4) + 4^3 + 8(8) + 8^3 + 8(12) + 12^3\right)$$

$$4(96 + 576 + 1824)$$

$$4(2496)$$

$$9984$$

2. Calculate $\lim_{n\to\infty} \sum_{k=1}^{n} (5 + k\frac{8}{n})^6 (\frac{4}{n})$ by evaluating the equivalent integral.

(10)

$$\lim_{n\to+\infty} \sum_{k=1}^{n} \left(5 + k\frac{8}{n}\right)^6 \frac{4}{n} = \lim_{n\to+\infty} \sum_{k=1}^{n} \frac{1}{2}\left(5 + k\frac{8}{n}\right)^6 \frac{8}{n}$$

$\Delta x = \frac{8}{n} \Rightarrow b - a = 8$

$a = 5$

$b = 13$

$$= \int_5^{13} \frac{1}{2} x^6 \, dx$$

$$= \frac{1}{2} \frac{x^7}{7} \Big|_5^{13} = \frac{1}{14}(13^7 - 5^7)$$

$= 4474956.57$

3. Find the area from $x = 0$ to $x = \pi$, between the x-axis and the curve $y = 2 - \cos(3x)$.

(10)

$$\int_0^{\pi} (2 - \cos(3x))\, dx = \left(2x - \frac{\sin(3x)}{3}\right)\Big|_0^{\pi}$$

$$= 2\pi - \frac{\sin(3\pi)}{3} - (0 - 0)$$

$$= 2\pi$$

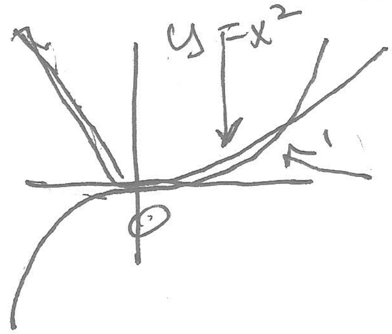
4. Find the average value of $f(x) = 2x + 4/x$ on the interval [7, 10].

$$\frac{1}{10-7} \int_7^{10} \left(2x + \frac{4}{x}\right) dx = \frac{1}{3}\left(x^2 + 4\ln x\right)\Big|_7^{10}$$

$$= \frac{1}{3}\left(10^2 + 4\ln 10 - (7^2 + 4\ln 7)\right)$$

$$= 17.476$$

5. Calculate the area bounded by the curves $y = x^2$ and $y = x^3$



10

$$\int_0^1 (x^2 - x^3)\,dx$$

$$\left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right)\Big|_0^1$$

$$\left(\frac{1}{3} - \frac{1}{4}\right) - (0)$$

$$\boxed{\frac{1}{12}}$$

6. A cylinder is 50 ft. tall with a radius of 4 feet is filled with water that weighs 64.5 lbs per cubic foot. How much work is required to empty the cylinder from the top?

10

volume $= \pi 4^2 \Delta h$

weight $= 62.5 \; 16\pi \Delta h$

work $= h \, 62.5 \; 16\pi \Delta x$

$$\int_0^{50} (62.5)(16\pi)\, h \, dh$$

$$(62.5)(16\pi)\frac{h^2}{2}\Big|_0^{50}$$

$$(62.5)(16\pi)\frac{50^2}{2} \text{ ft-lbs.}$$

3926990.82 ft-lbs

7. Find the derivative of $F(x) = \int_{-1}^{15x} \ln(\cos(6t) + 5)^{t}\, dt$

10

$y = \int_{-1}^{u} \ln(\cos(6t)+5)^t$ $\qquad u = 15x$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$= \ln(\cos(6u)+5)^u \,(15)$$

$$= 15 \ln(\cos(90x)+5)^{15x}$$

2

8. Calculate the following.

a. $\int \left(\frac{x}{1+x^4}\right) dx$

1 $\int \frac{x}{1+(x^2)^2}\,dx$

2 $u = x^2$

1 $\frac{du}{dx} = 2x$

1 $\frac{1}{2}du = x\,dx$

1 $\int \frac{\frac{1}{2}}{1+u^2}\,du$

2 $\frac{1}{2}\arctan u + C$

2 $\boxed{\frac{1}{2}\arctan x^2 + C}$

b. $\int \sin(x)\sec^2(\cos(x) + 3)dx$

2 $u = \cos x + 3$

2 $\frac{du}{dx} = -\sin x$

2 $-du = \sin x\,dx$

1 $-\int \sec^2 u\,du$

1 $-\tan u + C$

2 $\boxed{-\tan(\cos x + 3) + C}$

c. $\int_0^2 x^3(2x^4+3)^5\,dx$

2 $u = 2x^4 + 3$

2 $\frac{du}{dx} = 8x^3$

2 $\frac{1}{8}du = x^3\,dx$

2 $\int_3^{35} u^5 \frac{1}{8}\,du$

2 $\boxed{\frac{1}{8}(35^8 - 3^8)}$

2.8148×10^{11}