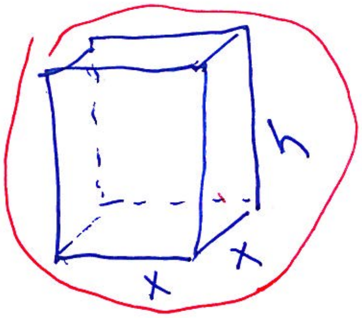
MAT 161 **Test 3** **Name** Key
Fall 2016
Instructions: To receive credit for all answers, show all work clearly in the space provided.

1. Find the dimensions of a 27 cubic foot box with an open-top and a square base, that is of minimal cost if the base costs \$2 per square foot and the vertical sides cost \$3 per square foot.

2



$\text{Min} = 2x^2 + 3(4xh)$ 2

subject to $x^2 h = 27$ 2

$h = 27x^{-2}$

2 $\text{Min } f(x) = 2x^2 + 12x(27x^{-2})$

on $(0, +\infty)$ $= 2x^2 + 12(27)x^{-1}$

2 $f'(x) = 4x - \dfrac{12(27)}{x^2} = 0$

$4x = \dfrac{12(27)}{x^2}$

$x^3 = 3(27)$

$x = 3\sqrt[3]{3}$ 4.3267

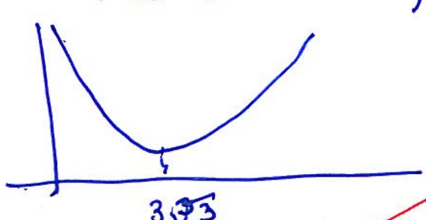
2 $f''(x) = 4 + \dfrac{2(12)(27)}{x^3}$

$f''(3\sqrt[3]{3}) > 0$

2 f has a rel. min at $x = 3\sqrt[3]{3}$

This is the only critical value

$\therefore$



f has an absolute min on $(0, +\infty)$ at $x = 3\sqrt[3]{3}$

1 $h = \dfrac{27}{9\sqrt[3]{9}} = \dfrac{3}{3^{2/3}} = 3^{\frac{1}{3}}$

$= 1.4422$

2. A population grows at a constant relative growth rate; at t= 0 hours, the population is 5 and at t = 6 hours the population is 12. Find the relative growth rate and determine the population when t = 9 hours

3

$$\frac{dP}{dt} = kP$$

$$P(t) = P_0 e^{kt}$$

3

$$5 = P(0) = P_0$$

$$P(t) = 5e^{kt}$$

3

$$12 = P(6) = 5e^{6k}$$

$$\frac{12}{5} = 2.4 = e^{6k}$$

$$\ln 2.4 = 6k$$

$$\frac{1}{6}\ln 2.4 = k \quad = .1459114562$$

3

$$P(t) = 5e^{\left(\frac{1}{6}\ln 2.4\right)t}$$

3

$$P(9) = 5e^{\frac{9}{6}\ln 2.4}$$

$$= \boxed{5(2.4)^{1.5}} \quad = 18.59032006$$

3. Calculate $\lim_{x\to 0} \dfrac{\ln(1+\sin(3x))}{4x}$. $\frac{0}{0}$

10

$$\lim_{x\to 0} \frac{\frac{1}{1+\sin(3x)} \cos(3x)(3)}{4} \quad 3 \;\; 3 = \frac{\frac{1}{1+0}(1)(3)}{4} \quad 2 \;\; 2 = \frac{3}{4}$$

4. Find f(x) if f "(x) = x^2 + cos(x), f '(0) = 3 and f(0) = 2 .

$f'(x) = \frac{1}{3}x^3 + \sin x + c$ 3

12

$3 = f'(0) = 0 + 0 + c$ 1

$f'(x) = \frac{1}{3}x^3 + \sin x + 3$ 2

$f(x) = \frac{1}{12}x^4 - \cos x + 3x + k$ 3

$2 = f(0) = 0 - 1 + 0 + k$ 1

$3 = k$

$\boxed{f(x) = \frac{1}{12}x^4 - \cos x + 3x + 3}$ 2

5. Use an initial guess of 1 and Newton's Method once to estimate the solution to $x^5 + 6x = 8$.

12

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^5 + 6x_n - 8}{5x_n^4 + 6} \quad 2 \;\; 2$$

2 1.

$x_0 = 1$ $\quad x_1 = 1 - \frac{1+6-8}{5+6} = 1 + \frac{1}{11} = \frac{12}{11}$

1.1. 1 2

7. For $f(x) = (x^2 - 4)^3$

a. Calculate the first and second derivative of *f(x)*.

$f'(x) = 3(x^2-4)^2 2x = 6x(x^2-4)^2$

6 $f''(x) = 6(x^2-4)2x(2x) + 3(x^2-4)^2 2$

b. Find the intervals where *f(x)* is increasing and decreasing.

$6(x^2-4)^2$ + + + x

6 x − − + x

− −2 − 0 + 2 +

f is decreasing on $(-\infty, 0)$

f is increasing on $(0, +\infty)$

c. Find the intervals where *f(x)* is concave up and concave down.

$f''(x) = 6(x^2-4)[4x^2 + (x^2-4)] = 6(x^2-4)(5x^2 - 4) = 30(x-2)(x+2)(x-\sqrt{.8})(x+\sqrt{.8})$

6 + − + − + ; −2, $-\sqrt{.8}$, $\sqrt{.8}$, +2

f is concave up on $(-\infty,-2) \cup (-\sqrt{.8}, \sqrt{.8}) \cup (2, +\infty)$

f is concave down on $(-2, -\sqrt{.8}) \cup (\sqrt{.8}, +2)$

d. Identify the local maximum, local minimum, and inflection points.

f has a local min at $x=0$

6 no local max

f has four inflection points at $x = \pm 2$, $\pm\sqrt{.8}$

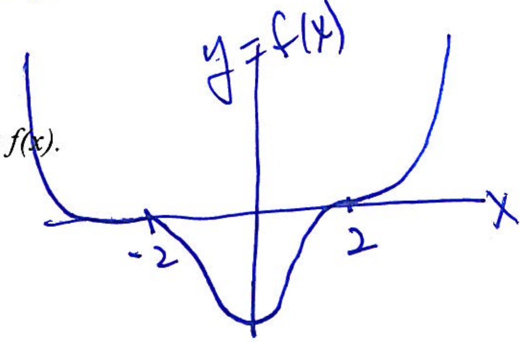
e. Find the x- and y-intercepts of $y = f(x)$.

$f(0) = -4^3 = -64$

6 $f(x) = 0 \Rightarrow x = \pm 2$

f. Sketch the graph of $y = f(x)$.

6



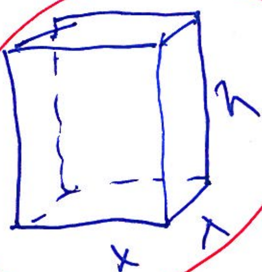
MAT 161 **Test 3** **Name** Key

Fall 2016 Version 2

Instructions: To receive credit for all answers, show all work clearly in the space provided.

1. Find the dimensions of a 27 cubic foot box with an open-top and a square base, that is of minimal cost if the base costs \$2 per square foot and the vertical sides cost \$6 per square foot.

2



Min $2x^2 + 6(4xh)$ 2

subject

$x^2 h = 27$

$h = 27x^{-2}$ 2

2 Min $f(x) = 2x^2 + 24(27)x^{-1}$

$f'(x) = 4x - \dfrac{24(27)}{x^2} = 0$

2 $x^3 = \dfrac{24(27)}{4} = 6(27) = 2(3^4)$

$x = 3\sqrt[3]{6}$ ≈ 5.45136

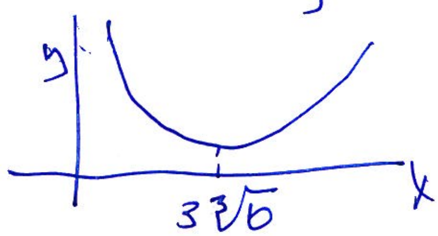
$f''(x) = 4 + \dfrac{2(24)(27)}{x^3}$

2 $f''(3\sqrt[3]{6}) > 0$

So By 2nd Test f has a rel. min at $x = 3\sqrt[3]{6}$

this is the only critical value

so



2 f(x) has a absolute min at $x = 3\sqrt[3]{6}$

$h = \dfrac{27}{3^2 \, 6^{2/3}}$

1 $= \dfrac{3^{\frac{1}{3}}}{2^{2/3}}$ $\approx .90856$

2. A population grows at a constant relative growth rate; at t= 0 hours, the population is 5 and at t = 6 hours the population is 14. Find the relative growth rate and determine the population when t = 9 hours

3 $\boxed{\frac{dP}{dt} = kP \qquad P(t) = P_0 e^{kt}}$

3 $\boxed{P_0 = 5 \qquad P(t) = 5e^{kt}}$

3 $\boxed{14 = P(6) = 5e^{6k} \qquad 2.8 = e^{6k} \qquad \ln 2.8 = 6k \qquad \frac{1}{6}\ln 2.8 = k}$ .1716032362

3 $\boxed{P(t) = 5e^{\frac{t}{6}\ln 2.8}}$

3 $\boxed{P(9) = 5e^{\frac{9}{6}\ln 2.8} = 5\boxed{2.8^{1.5}}}$ 23.42648074

3. Calculate $\lim_{x\to 0} \dfrac{\ln(\cos(3x))}{4x}$. $\frac{0}{0}$

10

$$\lim_{x\to 0} \frac{\frac{1}{\cos(3x)} - \sin(3x)(3)}{4} \; 3 \quad = \quad \frac{\frac{1}{1}(-0)\,3}{4} \; 2 = 0$$

3 2

4. Find f(x) if f "(x) = x^2 + sin(x), f '(0) = 3 and f(0) = 2 .

12

$f'(x) = \frac{1}{3}x^3 - \cos x + C$ 3

$3 = f'(0) = 0 - 1 + C \Rightarrow C = 4$ 1

$f'(x) = \frac{1}{3}x^3 - \cos x + 4$ 2

$f(x) = \frac{1}{12}x^4 - \sin x + 4x + k$ 3

$2 = f(0) = 0 + k$ 1

$k = 2$

$f(x) = \frac{1}{12}x^4 - \sin x + 4x + 2$ 2

5. Use an initial guess of 1 and Newton's Method once to estimate the solution to $x^5 + 6x = 6$.

12

$f(x) = x^5 + 6x - 6 = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^5 + 6x_n - 6}{5x_n^4 + 6}$$ 2 2 1

$$= 1 - \frac{1+6-6}{5+6} = 1 - \frac{1}{11} = \frac{10}{11}$$

1 1 1 2

7. For $f(x) = (x^2 - 9)^3$ $= (x+3)^3(x-3)^3$

a. Calculate the first and second derivative of $f(x)$.

6

$f'(x) = 3(x^2-9)^2(2x) = 6x(x^2-9)^2$

$f''(x) = 6(x^2-9)^2 + 6x\, 2(x^2-9)2x$

$= 6(x^2-9)[x^2-9+4x^2]$

$= 6(x^2-9)(5x^2-9)$

b. Find the intervals where $f(x)$ is increasing and decreasing.

6

$(x^2-9)^2$: + + + +

$6x$: − − + +

− −3 − 0 + 3 +

f is dec on $(-\infty, 0)$

f is inc on $(0, +\infty)$

c. Find the intervals where $f(x)$ is concave up and concave down.

6

+ − + − +

-3 $\frac{-3}{\sqrt{5}}$ $\frac{3}{\sqrt{5}}$ 3

f is concave up on $(-\infty, -3) \cup (-\frac{3}{\sqrt{5}}, \frac{3}{\sqrt{5}}) \cup (3, +\infty)$

f is concave down on $(-3, -\frac{3}{\sqrt{5}}) \cup (\frac{3}{\sqrt{5}}, +\infty)$

d. Identify the local maximum, local minimum, and inflection points.

6

local min at $x = 0$

no local max

inflection points at $x = \pm 3, \pm \frac{3}{\sqrt{5}}$

e. Find the x- and y-intercepts of $y = f(x)$.

6

$f(0) = \cancel{0} -9^3$

$f(x) = 0 \Rightarrow x = 0, \pm 3$

f. Sketch the graph of $y = f(x)$.

6