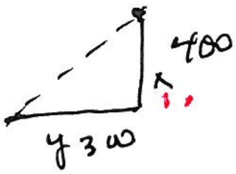
MAT 161 Test 2 Name Key / Record
Fall 2016 Version 1

For complete credit, show all work.

1. At noon Air Trump is 400 miles north of Wilmington traveling south at 300 miles per hour while Air Clinton is 300 miles west of Wilmington traveling east at 230 miles per hour. At what rate is the distance between the two planes changing at noon.



$\frac{dy}{dt} = -300$

$\frac{dx}{dt} = -230$

14

$z^2 = x^2 + y^2$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$

$= \frac{400(-300) + 300(-230)}{100\sqrt{4^2+3^2}} = \frac{400(-300)+300(-230)}{500}$

$= .8(-300) + .6(-230)$

$= -240 - 138$

$= \boxed{-378 \text{ mph}}$

2. Calculate the linearization of $f(x) = x^{\frac{7}{3}}$ at $a = 8$ Then estimate $(8.03)^{\frac{7}{3}}$ using the linearization.

$f(x) = x^{7/3}$; $f'(x) = \frac{7}{3}x^{4/3}$

$f(8) = 8^{7/3} = 2^7 = 128$; $f'(8) = \frac{7}{3}8^{4/3} = \frac{7(16)}{3}$

13

Exact $8.03^{7/3} \approx 129.1228012$

$L(x) = 128 + \frac{7}{3}(16)(x-8)$

$L(8.03) = 128 + \frac{7}{3}(16)(8.03-8)$ 2

$= 128 + \frac{7}{3}(16)(.03) = 128 + 1.12 = \boxed{129.12}$ 2

3. A spherical balloon is being filled with water at the rate of 2 cubic centimeters per second. At what rate is the radius increasing when the radius is 5 centimeters. Note that the volume of a sphere is $V = \frac{4}{3}\pi r^3$ where r is the radius of the ballon.

13

2 $\frac{dV}{dt} = 2$

2 $\frac{dr}{dt} = ?$ when $r = 5$

2 $V = \frac{4}{3}\pi r^3$

2 $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

2 $2 = 4\pi(5^2)\frac{dr}{dt}$

3 $\frac{.02}{\pi} = \frac{dr}{dt} = .0064$ cm/second

In problems 5-10, calculate the derivative of y with respect to x.
10 apiece

4. $f(x) = \tan(x)\sqrt{1+5x^3} + 7x + 3.$

$f'(x) = \underbrace{\sec^2 x}_{2} \underbrace{\sqrt{1+5x^3}}_{1} \underbrace{+}_{1} \underbrace{\tan x}_{1} \underbrace{\tfrac{1}{2}(1+5x^3)^{-\frac{1}{2}}(15x^2)}_{3} \underbrace{+7}_{1} \underbrace{+0}_{1}$

5. $y = \dfrac{\log(5x+e^x)}{6^{8x}}$

$\dfrac{dy}{dx} = \dfrac{\overset{1}{6^{8x}} \overbrace{\dfrac{1}{(5x+e^x)\ln 10}(5+e^x)}^{2} \overset{1}{-} \overbrace{\log(5x+e^x)}^{1} \overbrace{(6^{8x}\ln 6\; 8)}^{2}}{(6^{8x})^2}$ 3

6. $y = \arcsin(\cos(x)) + \tan(x)$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-(\cos(x))^2 \cancel{+\tan x)^2}}}$ 2 4 $\overset{2}{(-\sin x)} + \overset{3}{\sec^2 x}$

7. $(y^2 - 5xy) + \sin(3xy) = \ln(7x + y)$

$2y\overset{1}{\dfrac{dy}{dx}} - \overset{1}{5y} - \overset{1}{5x\dfrac{dy}{dx}} + \overset{1}{\cos(3xy)}\overset{1}{(3y} + \overset{1}{3x\dfrac{dy}{dx}}) = \overset{1}{\dfrac{1}{7x+y}}\overset{1}{(7+\dfrac{dy}{dx})}$

1 $\left(2y - 5x + 3x\cos(3xy) - \dfrac{1}{7x+y}\right)\dfrac{dy}{dx} = \dfrac{7}{7x+y} + 5y - 3y\cos(3xy)$

$\boxed{\dfrac{dy}{dx} = \dfrac{\dfrac{7}{7x+y} + 5y - 3y\cos(3xy)}{2y - 5x + 3x\cos(3xy) - \dfrac{1}{7x+y}}}$ 1

8. $y = \sec(5x) - \cos(x^2)$

$\dfrac{dy}{dx} = \underbrace{\sec(5x)\tan(5x)}_{2\ 2}\underbrace{(5)}_{2} \underbrace{+}_{1} \underbrace{\sin(x^2)}_{2}\underbrace{2x}_{1}$

9. $y = (1 + x^2)^{\tan(3x)}$ $\quad \ln y = \tan(3x)\ln(1+x^2)$ 1

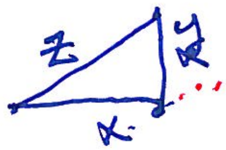
$\dfrac{1}{y}\dfrac{dy}{dx} \overset{2}{=} \overset{1}{\sec^2(3x)}\,\overset{1}{3}\,\overset{1}{\ln(1+x^2)} \overset{1}{+} \overset{1}{\tan(3x)\dfrac{2x}{1+x^2}}$

$\dfrac{dy}{dx} = \overset{1}{(1+x^2)^{\tan(3x)}}\left(\sec^2(3x)\, 3\ln(1+x^2) + \overset{1}{\tan(3x)\dfrac{2x}{1+x^2}}\right)$

MAT 161 Test 2 Name Key
Fall 2016 Version 2

For complete credit, show all work.

1. At noon Air Trump is 400 miles north of Wilmington traveling north at 300 miles per hour while Air Clinton is 300 miles west of Wilmington traveling east at 230 miles per hour. At what rate is the distance between the two planes changing at noon.



$y = 400$ $\frac{dy}{dt} = 300$

$x = 300$ $\frac{dx}{dt} = -230$

(14)

$z^2 = x^2 + y^2$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z} = \frac{300(-230) + 400(300)}{\sqrt{(400)^2 + (300)^2}}$

$= \frac{3(-230) + 4(300)}{5} = 102$ miles per hour

2. Calculate the linearization of $f(x) = x^{\frac{4}{3}}$ at $a = 8$ Then estimate $(8.03)^{\frac{4}{3}}$ using the linearization.

$f(x) = x^{4/3}$; $f'(x) = \frac{4}{3}x^{\frac{1}{3}}$

$f(8) = 8^{4/3} = 16$; $f'(8) = \frac{4}{3}8^{\frac{1}{3}} = \frac{8}{3}$

(13)

$L(x) = 16 + \frac{8}{3}(x-8)$ 2

$L(8.03) = 16 + \frac{8}{3}(.03) = 16 + .08 = 16.08$ 2

$8.03^{4/3}$ exact $= 16.08004996$

3. A spherical balloon is being filled with water at the rate of 3 cubic centimeters per second. At what rate is the radius increasing when the radius is 10 centimeters. Note that the volume of a sphere is $V = \frac{4}{3}\pi r^3$ where r is the radius of the ballon.

(13)

2 $\frac{dV}{dt} = 3$

2 $\frac{dr}{dt} = ?$ when $r = 10$

2 $V = \frac{4}{3}\pi r^3$

2 $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

2 $3 = 4\pi(10)^2 \frac{dr}{dt}$

3 $\frac{dr}{dt} = \frac{3}{4\pi(10^2)} = .002387$ cm/second

In problems 5-10, calculate the derivative of y with respect to x.

4. $f(x) = \tan(x) + \sqrt{1+5x^3}\,(7x+3)$.

$f'(x) = \sec^2 x + \frac{1}{2}(1+5x^2)^{-\frac{1}{2}} 15x^2 (7x+3) + (1+5x^3)^{\frac{1}{2}}(7)$

2 2 2 2 2

5. $y = \dfrac{\log(5x+e^x)}{8^{6x}}$

2 1 1 2

$\frac{dy}{dx} = \dfrac{(8^{6x})\left(\frac{1}{5x+e^x}\right)(\ln 10)(5+e^x) - \log(5x+e^x)\, 8^{6x}(\ln 8)6}{(8^{6x})^2}$ 3

6. $y = \arcsin(\cos(x)) + \tan(x)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-\cos^2 x}}(-\sin x) + \sec^2 x$

4 3 3

7. $(y^2 - 5xy) + \ln(3xy) = \sin(7x+y)$

$2y\frac{dy}{dx} - 5y - 5x\frac{dy}{dx} + \frac{1}{3xy}\left\{3y + 3x\frac{dy}{dx}\right\} = \cos(7x+y)\left(7+\frac{dy}{dx}\right)$

$\frac{dy}{dx} = \dfrac{5y - \frac{3y}{3xy} + 7\cos(7x+y)}{2y - 5x + \frac{3x}{3xy} - \cos(7x+y)}$ 1 1

8. $y = \cos(5x) - \sec(x^2)$

$\frac{dy}{dx} = -\sin(5x)\,5 - \sec(x^2)\tan x^2\,(2x)$

2 2 2 2 2

9. $y = (1+x^2)^{\tan(3x)}$

$\ln y = \tan(3x)\ln(1+x^2)$

2 2 2

$\frac{1}{y}\frac{dy}{dx} = \sec^2(3x)\,3\ln(1+x^2) + \tan 3x\,\frac{2x}{1+x^2}$

$\frac{dy}{dx} = (1+x^2)^{\tan(3x)}\,;$ 1