

Instructions: To receive credit for all answers, show all work clearly in the space provided. You may use graphing calculators. This is designed to be a 50 minute test.

1. Find the indicated limits. If the limit does not exist, tell why.

a. $\lim_{x \rightarrow 7} \frac{x^2 - x - 42}{x^2 - 49}$

b. $\lim_{x \rightarrow -7^+} \frac{x^2 - x - 42}{x^2 - 49}$

c. $\lim_{x \rightarrow 10} \frac{x^2 - x - 42}{x^2 - 49}$

d. $\lim_{x \rightarrow +\infty} \frac{x^2 - x - 42}{x^2 - 49}$

2. a. Suppose that for all real numbers x , $x \leq f(x) \leq .25x^2 + 1$. Is $f(x)$ continuous at 1? Why or why not?

b. Is the function $f(x)$ in question 2a continuous at $x = 2$? Why or why not?

3. Using the precise (δ, ε) definition of limits, prove that $\lim_{x \rightarrow 4} (3x + 8) = 20$
4. Suppose $f(x) > 0$ all real numbers less than 4 as a domain and the graph of $y = f(x)$ has a vertical asymptote at $x = 4$.
- a. What is $\lim_{x \rightarrow 4^-} f(x)$?
- b. Can you calculate $\lim_{x \rightarrow 4^+} f(x)$? Why or why not?
5. Use the Intermediate Value Theorem to find an interval where there is a solution to the equation $6 = 3x + 3x^2 + x^3$ in the interval.

6. a. Use the definition of a derivative to find $f'(x)$ where $f(x) = 5x^2 - 3x$.

b. Find the equation of the tangent line to the graph of $y = f(x)$ at $(1, 2)$.

c. Find the instantaneous rate of change of $y = f(x)$ with respect to x when $x = 4$.

d. Find the average rate of change of $y = f(x)$ with respect to x over the interval $[2, 4]$?

7. Find $f'(x)$ if $f(x) = \frac{1+5x}{3+2x}$.