MAT 161 **Test 1** **Name** Key / Record

Fall 2016 Version 2

Instructions: To receive credit for all answers, show all work clearly in the space provided. You may use graphing calculators. This is designed to be a 50 minute test.

1. Find the indicated limits. If the limit does not exist, tell why.

20

5 each

a. $\lim_{x\to 7} \frac{x^2-x-42}{x^2-49}$

$\lim_{x\to 7} \frac{(x-7)(x+6)}{(x-7)(x+7)}$

$\frac{7+6}{7+7} = \frac{13}{14}$

3 for correct answer

2 for reasoning

b. $\lim_{x\to -7^+} \frac{x^2-x-42}{x^2-49}$

$\lim_{x\to -7^+} \frac{x+6}{x+7}$

$= -\infty$

$-7 < x < -6$

>0

$\therefore x+7$ and $x+6 < 0$

$\frac{x+7}{x+6} < 0$

c. $\lim_{x\to 10} \frac{x^2-x-42}{x^2-49}$

$= \frac{10^2-10-42}{10^2-49}$

$= \frac{48}{51} = \frac{16}{17}$

d. $\lim_{x\to +\infty} \frac{x^2-x-42}{x^2-49} = \frac{1}{1} = 1$

by leading coefficients

2. a. Suppose that for all real numbers x, $x \le f(x) \le .25x^2+1$, Is f(x) continuous at 1 ? Why or why not?

5

$1 \le f(1) \le .25(1^2)+1$

$\therefore 1 \le f(1) \le 1.25$ 3

we cannot determine f(1). 1

$\therefore$ there is insufficient evidence to determine if f(x) is cont at x = 1 1

b. Is the function f(x) in question 2a continuous at x = 2? Why or why not?

5

$2 \le f(2) \le .25(2^2)+1 = 1+1$ 2

$\therefore f(2) = 2$

and $\lim_{x\to 2} x = 2$ and $\lim_{x\to 2} .25x^2+1 = 2$ 1

$\therefore$ By squeeze law, $\lim_{x\to 2} f(x) = 2$ 1

$\therefore$ f is cont at x = 2. 1

/30

3. Using the precise (δ, ε) definition of limits, prove that $\lim\limits_{x \to 4} (3x+8) = 20$

Given $\epsilon > 0$ choose $\delta = \frac{\epsilon}{3}$ 2

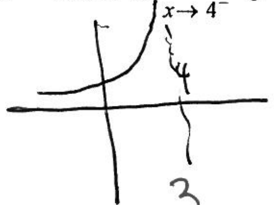
(10) Then if $0 < |x-4| < \delta$ 1 → $\cancel{|3x+8-4| < \epsilon}$

$|x-4| < \epsilon/3$ 1 $\quad |3x+8-12-8| < \epsilon$ 1

$|3x-12| < \epsilon$ 1 $\quad |(3x+8)-20| < \epsilon$ 2

4. Suppose f(x) > 0 for all real numbers less than 4 as a domain and the graph of y = f(x) has a vertical asymptote at x = 4.

5 a. What is $\lim\limits_{x \to 4^-} f(x)$?



$\lim\limits_{x \to 4^-} f(x) = +\infty$

3 2

5 b. Can you calculate $\lim\limits_{x \to 4^+} f(x)$? Why or why not?

3 f(x) is not defined if $x > 4$

2 ∴ $\lim\limits_{x \to 4^+} f(x)$ cannot be calculated

5. Use the Intermediate Value Theorem to find an interval where there is a solution to the equation

10 $6 = 3x + 3x^2 + x^3$ in the interval.

$6 = 3x + 3x^2 + x^3$ is equivalent to $0 = 3x + 3x^2 + x^3 - 6$ 2

Let $f(x) = 3x + 3x^2 + x^3 - 6$ which is cont 2

$f(0) = -6$ 1

$f(1) = 3+3+1-6 = 1$ 1

0 is between -6 & 1 2

∴ there is a z between 0 & 1 so that $f(z) = 0$. 2

6. a. Use the definition of a derivative to find $f'(x)$ where $f(x) = 5x^2 - 3x$.

8

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad 2$$

$$= \lim_{h\to 0} \frac{5(x+h)^2 - 3(x+h) - (5x^2-3x)}{h} \quad 2$$

8

$$= \lim_{h\to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 - \cancel{3x} - 3h - \cancel{5x^2} + \cancel{3x}}{h} \quad 1$$

$$= \lim_{h\to 0} 10x + 5h - 3 = 10x - 3$$

1 2

b. Find the equation of the tangent line to the graph of y = f(x) at (1, 2) .

8

$f'(1) = 10 - 3 = 7$ 2

$y - 2 = 7(x-1)$ 3

$y - 2 = 7x - 7$ 1

$y = 7x - 5$ 2

c. Find the instantaneous rate of change of y = f(x) with respect to x when x = 4.

$f'(4) = 10(4) - 3 = 40 - 3 = 37$

8 4 4

d. Find the average rate of change of y = f(x) with respect to x over the interval [2 , 4] ?

8

$$\frac{f(4)-f(2)}{4-2} = \frac{5(4^2) - 3(4) - (5\cdot 2^2 - 3(2))}{2} = \frac{68-14}{2} = \frac{54}{2} = 27$$

4 2 2

7. Find $f'(x)$ if $f(x) = \dfrac{1+5x}{3+2x}$.

8

$$f'(x) = \lim_{z\to x} \frac{f(z) - f(x)}{z - x} = \lim_{z\to x} \frac{\frac{1+5z}{3+2z} - \frac{1+5x}{3+2x}}{z-x} \quad 2 \quad 2$$

$$= \lim_{z\to x} \frac{(1+5z)(3+2x) - (3+2z)(1+5x)}{(3+2z)(3+2x)(z-x)} \quad 1$$

$$= \lim_{z\to x} \frac{\cancel{3} + 15z + 2x + \cancel{10xz} - (\cancel{3} + 2z + 15x + \cancel{10xz})}{(z-x)(3+2z)(3+2x)} \quad 1$$

$$= \lim_{z\to x} \frac{13z - 13x}{(z-x)(3+2z)(3+2x)} = \frac{13}{(3+2x)^2}$$

1 1

/40