MAT 161 **Test 1** **Name** Key / Record

Fall 2016 version 1

Instructions: To receive credit for all answers, show all work clearly in the space provided. You may use graphing calculators. This is designed to be a 50 minute test.

1. Find the indicated limits. If the limit does not exist, tell why.

20
5 each

a. $\lim_{x \to -7} \frac{x^2+x-42}{x^2-49}$

3 for correct answer
2 reasoning

$\lim_{x \to -7} \frac{(x+7)(x-6)}{(x+7)(x-7)}$

$\lim_{x \to -7} \frac{x-6}{x-7} = \frac{-13}{-14} = \frac{13}{14}$

$= .92857$

b. $\lim_{x \to 7^-} \frac{x^2+x-42}{x^2-49}$

$6 < x < 7$

$x - 6 > 0$

$x - 7 < 0$

$\therefore \frac{x-6}{x-7} < 0$

$\lim_{x \to 7^-} \frac{x-6}{x-7} = -\infty$

doesn't exist
-1

c. $\lim_{x \to 10} \frac{x^2+x-42}{x^2-49}$

$\frac{10^2+10-42}{10^2-49} = \frac{68}{51}$

$= \frac{4}{3}$

d. $\lim_{x \to +\infty} \frac{x^2+x-42}{x^2-49}$

$= \frac{1}{1} = 1$ by leading coeff.

2. a. Suppose that for all real numbers x, $4(x-1) \le f(x) \le x^2$, Is f(x) continuous at 1 ? Why or why not?

5

at $x=1$, $4(1-1) = 0 \le f(1) \le 1 = 1^2$. 3

There is insufficient evidence to determine if f is cont. at 1. 1

We cannot determine the value of f(1) 1

b. Is the function f(x) in question 2a continuous at x = 2? Why or why not?

5 at $x=2$ $4(2-1) = 4 \le f(2) = 4 = 2^2$ 2

$\therefore f(2) = 4$

Furthermore $\lim_{x \to 2} f(x) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 4$ 1

$\therefore \lim_{x \to 2} f(x) = f(2)$ 1

$\therefore$ f is cont at $x = 2$. 1

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3. Using the precise (δ, ε) definition of limits, prove that $\lim_{x \to 3} (5x+2) = 17$

2 2

(10) Given $\epsilon > 0$ choose $\delta = \epsilon/5$.

Then if $0 < |x-3| < \delta$ 1 → $|5x-15+2-2| < \epsilon$ 1

$|x-3| < \epsilon/5$ 1 $|5x+2-17| < \epsilon$ 1

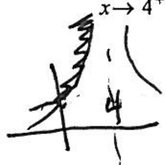
$|5x-15| < \epsilon$ 1

$|5x+2-17| < \epsilon$ 1

4. Suppose f(x) > 0 for all real numbers greater than 4 as a domain and the graph of y = f(x) has a vertical asymptote at x = 4.

a. What is $\lim_{x \to 4^+} f(x)$?

(5)



$\lim_{x \to 4^+} f(x) = +\infty$

2

3

b. Can you calculate $\lim_{x \to 4^-} f(x)$? Why or why not?

(5) $f(x)$ for $x < 4$ is not defined. 3

We cannot determine $\lim_{x \to 4^+} f(x)$. 2

5. Use the Intermediate Value Theorem to find an interval where there is a solution to the equation $6 = 6x - x^3$ in the interval.

(10) 2 Solving $6 = 6x - x^3$ is equivalent to solving $0 = 6x - x^3 - 6$. which is cont.

2 Let $f(x) = 6x - x^3 - 6$

1 $f(0) = -6$

1 ~~$f(2) = 6 \cdot 2 - 2^3 - 6 = 2$~~ $f(-3) = 3$

2 0 is between -6 & 3

∴ There is z between 0 and -3 so that

2 $f(z) = 0$.

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6. a. Use the definition of a derivative to find $f'(x)$ where $f(x) = 3x^2 - 5x$.

(8)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ 2

$$= \lim_{h\to 0} \frac{3(x+h)^2 - 5(x+h) - (3x^2-5x)}{h}$$ 2

$$= \lim_{h\to 0} \frac{\cancel{3x^2}+6xh+3h^2-\cancel{5x}-5h-\cancel{3x^2}+\cancel{5x}}{h}$$ 1

$$= \lim_{h\to 0} 6x+3h-5 = \boxed{6x-5}$$

1 2

b. Find the equation of the tangent line to the graph of y = f(x) at (1, -2) .

(8)

$f'(1) = 6(1)-5 = 1$ 2

$y-(-2) = 1(x-1)$ 3

$y+2 = x-1$ 1

$y = x-3$ 2

c. Find the instantaneous rate of change of y = f(x) with respect to x when x = 4.

(8)

$f'(4) = 6(4)-5 = 24-5 = \boxed{19}$

4 4

d. Find the average rate of change of y = f(x) with respect to x over the interval [2 , 4] ?

(8)

$$\frac{f(4)-f(2)}{4-2} = \frac{3(4^2)-5(4) - (3\cdot 2^2 - 5(2))}{2} = \frac{28-2}{2} = 14-1 = \boxed{13}$$

4 2 2

7. Find $f'(x)$ if $f(x) = \dfrac{3+2x}{1+5x}$.

(8)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left(\frac{3+2(x+h)}{1+5(x+h)} - \frac{3+2x}{1+5x}\right)}{h}$$ 2 1

$$= \lim_{h\to 0} \frac{(3+2(x+h))(1+5x) - (3+2x)(1+5(x+h))}{(1+5(x+h))(1+5x)h}$$ 1

$$= \lim_{h\to 0} \frac{(3+2x+2h)(1+5x) - (3+2x)((1+5x)+5h)}{h(1+5(x+h))(1+5x)}$$ 1

$$= \lim_{h\to 0} \frac{2h(1+5x) - (3+2x)5h}{h(1+5(x+h))(1+5x)} = \frac{2(1+5x)-5(3+2x)}{(1+5x)^2} = \frac{-13}{(1+5x)^2}$$

1 1 1

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