Chapter 2 Review Problems

(4) $\lim_{x \to 3} \frac{x^2-9}{x^2+2x-3} = \frac{3^2-9}{3^2+2(3)-3} = \frac{0}{12} = \boxed{0}$

(8) $\lim_{t \to 2} \frac{t^2-4}{t^3-8} = \lim_{t \to 2} \frac{\cancel{(t-2)}(t+2)}{\cancel{(t-2)}(t^2+2t+4)} = \frac{2+2}{2^2+2(2)+4} = \frac{4}{12} = \boxed{\frac{1}{3}}$

(12) $\lim_{x \to 3} \frac{\sqrt{x+6}-x}{x^3-3x^2} = \lim_{x \to 3} \frac{\sqrt{x+6}-x}{x^2(x-3)} \cdot \frac{\sqrt{x+6}+x}{\sqrt{x+6}+x}$

$= \lim_{x \to 3} \frac{x+6-x^2}{x^2(x-3)(\sqrt{x+6}+x)} = \lim_{x \to 3} \frac{-(x-3)(x+2)}{x^2(x-3)(\sqrt{x+6}+x)}$

$= \lim_{x \to 3} \frac{-(x+2)}{x^2(\sqrt{x+6}+x)} = \frac{-(3+2)}{9(\sqrt{9}+3)}$

$= \frac{-5}{9(6)} = \boxed{\frac{-5}{54}}$

(16) $\lim_{x \to -\infty} \frac{1-2x^2-x^4}{5+x-3x^4} = +\frac{1}{3}$ or $\frac{-1}{-3}$ by leading coefficients

(17) $\lim_{x \to +\infty} \left(\sqrt{x^2+4x+1}-x\right) = \lim_{x \to +\infty} \frac{\cancel{x^2}+4x+1-\cancel{x^2}}{\sqrt{x^2+4x+1}+x}$

$= \lim_{x \to +\infty} \frac{4+\frac{1}{x}}{\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}+\frac{1}{x}} = \frac{4}{\sqrt{1}} = \boxed{4}$

(19) $\lim_{x \to 0^+} \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$, since $\lim_{x \to 0^+} \frac{1}{x} = +\infty$ and $\lim_{x \to +\infty} \tan^{-1}(x) = \frac{\pi}{2}$

(20) $\lim_{x \to 1} \left(\frac{1}{x-1} + \frac{1}{x^2-3x+2}\right) = \lim_{x \to 1} \left(\frac{x-2}{(x-1)(x-2)} + \frac{1}{(x-1)(x-2)}\right) = \lim_{x \to 1} \frac{x-1}{(x-1)(x-2)}$

$= \lim_{x \to 1} \frac{1}{x-2} = \frac{1}{1-2} = \frac{1}{-1} = \boxed{-1}$

(23) $\lim_{x\to 1} 2x-1 = 2(1)-1=1 = \lim_{x\to 1} x^2$

and $2x-1 \le f(x) \le x^2$ for $0<x<3 \Rightarrow$ ∴ By Squeeze Law $\lim_{x\to 1} f(x) = 1$

(25) Given $\epsilon>0$ choose $\delta = \frac{\epsilon}{5}$.

Then if $0<|x-2|<\delta = \epsilon/5$

$|5x-10| < \epsilon$

$|10-5x| < \epsilon \Rightarrow |(14-5x)-4| < \epsilon.$

(34) Let $f(x) = \cos\sqrt{x} - (e^x-2)$

$f(0) = \cos 0 - (1-2) = 1-(1-2) = 2 > 0$

$f(1) = \cos\sqrt{1} - (e^1-2) = \cos 1 + 2 - e = -.17798 < 0$

∴ $\boxed{\text{there is an } x \in (0,1) \text{ so that } f(x)=0}$

(35) Let $f(x) = 9-2x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{9-2(x+h)^2-(9-2x^2)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2-(x+h)^2)}{h} = \lim_{h\to 0} \frac{2(x-(x+h))(x+(x+h))}{h}$$

$$= \lim_{h\to 0} \frac{-2(x+x+h)}{1} = -2(2x) = -4x$$

(a) At $(2,1)$, $f'(2) = -4(2) = -8$

(b) $y-1 = -8(x-2)$

$y-1 = -8x+16$

$\boxed{y = -8x+17}$

(40) If $f'(a) = \lim_{h \to 0} \frac{(2+h)^6 - 64}{h}$, then $a = 2$ and $f(x) = x^6$

so that $f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$.

(45) $f(x) = \sqrt{3 - 5x}$

(a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\sqrt{3 - 5(x+h)} - \sqrt{3-5x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{3-5x-5h} - \sqrt{3-5x}}{h} \cdot \frac{\sqrt{3-5x-5h} + \sqrt{3-5x}}{\sqrt{3-5x-5h} + \sqrt{3-5x}}$$

$$= \lim_{h \to 0} \frac{\cancel{3} - \cancel{5x} - 5h - (\cancel{3} - \cancel{5x})}{h\left(\sqrt{3-5x-3h} + \sqrt{3-5x}\right)}$$

$$= \frac{-5}{2\sqrt{3-5x}}$$

(b) domain of $f(x)$

$3 - 5x \geq 0$

$3 \geq 5x$

$.6 \geq x$

$(-\infty, +.6]$

domain of $f'(x)$

$3 - 5x > 0$

$3 > 5x$

$.6 > x$

$(-\infty, +.6)$

37) $s(t) = 1 + 2t + \frac{1}{4}t^2$

(a) $\dfrac{s(t)-s(1)}{t-1} = \dfrac{\cancel{1} + 2t + \frac{1}{4}t^2 - (\cancel{1} + 2 + \frac{1}{4})}{t-1} = \dfrac{2(t-1) + \frac{1}{4}(t^2-1)}{t-1}$

$= \dfrac{2(t-1) + \frac{1}{4}(t-1)(t+1)}{t-1} = 2 + \frac{1}{4}(t+1)$

(i) $t = 3$ $\quad \dfrac{s(t)-s(1)}{t-1} = 2 + \frac{1}{4}(3+1)$

(ii) $t = 2$ $\quad 2 + \frac{1}{4}(2+1)$

(iii) $t = 1.5$ $\quad 2 + \frac{1}{4}(1.5+1)$

(iv) $t = 1.1$ $\quad 2 + \frac{1}{4}(1.1+1)$

(b) $s'(1) = \lim\limits_{t\to 1} \dfrac{s(t)-s(1)}{t-1} = 2 + \frac{1}{4}(1+1) = \boxed{2.5}$

39 (a) $f(x) = x^3 - 2x$

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{(x+h)^3 - 2(x+h) - [x^3 - 2x]}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{2x} - 2h - \cancel{x^3} + \cancel{2x}}{h}$

$= \lim\limits_{h\to 0} (3x^2 + 3xh - 2) = 3x^2 - 2$

$f'(2) = 3(2^2) - 2 = 12 - 2 = 10$

(b) $y - 4 = 10(x-2)$

$y - 4 = 10x - 20$

$\boxed{y = 10x + 16}$