MAT 161 Final Exam Name _____
Wednesday, Dec. 12, 2012

1. Calculate the following limits:

a. $\lim_{x\to 5} \dfrac{7x^2-33x-10}{3x^2-14x-15}$

b. $\lim_{x\to+\infty} \dfrac{12x^2}{3x+2}-4x$

2. Use the definition of the derivative to find $f'(x)$ when $f(x) = 5x^2+3x+7$.

3. Give a delta-epsilon argument that $\lim_{x\to 4} 2x+1=9.$

4. Tell why f(x) = $|x+7|$ is not differentiable at $x = -7$ but is continuous there.

5. Calculate $\frac{dy}{dx}$ for the following:

a. $y = \left[x^3 e^{2x} + \sin(2x)\right]^5$

b. $y = \dfrac{\tan(3x) + 5x^2 + 1}{2 + \sin(x^2)}$

c. $y = \tan^{-1}(5^{2x})$

d. $\cos(y) = (\sec(x+3))^{\tan(y)}$

e. $x^3 + y\sin x = y^2$

f. $y = \displaystyle\int_0^{e^{4x}} \csc(t^2 + 2)\,dt$

6. Use a linear approximation to estimate $8100.5^{\frac{1}{2}}$

7. Find the equation of the tangent line to the curve $y = 2x^6 - 9x^7$ at (1,-7).

8. Water is pumped at the rate of .01 cubic yard per minute from a coneof height 2 yards and of radius 3 yards at the base. How fast is the depth of the water falling in the cone when the depth is .5 yards?

[For additional extra credit: Suppose the bottom 1/3 (in volume) of the cone is filled with a fluid weighing 1800 lbs per cubic yard. How much work is done in empting the cone by pumping the fluid to a valve at the top of the sphere.]

9. What are the dimensions of a cylinder of maximum volume if the surface area is to be 20 square meters?

10. Calculate

a. $\int_0^1 \left(x^2 + (2x+1)^3\right) dx$

b. $\int \frac{\sin 4x}{(1+\cos 4x)^{5.}}\, dx$

11. For $f(x) = 6x^5 - 10x^{34}$

a. Calculate the first and second derivative of *f(x).*

b. Find the intervals where *f(x)* is increasing and decreasing.

c. Find the intervals where *f(x)* is concave up and concave down.

d. Identify the local maximum, local minimum, and inflection points.

e. Find the *x*- and *y*-intercepts of *y = f(x).*

f. Sketch the graph of *y = f(x).*

12. The volume of cube is estimated to be 1000 +/- .02 cubic cm. Estimate the edge and the measurement error for the edge.

13. Find the area from $x = 0$ to $x = 5$ between the x-axis and the curve $y = 3x + x^3$.

14. Evaluate the Riemann Sum for $f(x) = 4x + 2$, $5 \le x \le 8$, with six subintervals, taking the sample points to be midpoints.

15. Find $f(x)$ if $f''(x) = 3x^2 - 6x$ and $f(1) = 10$ and $f'(1) = 2$.

16. Find the average value of $f(x) = x^4$ on [1 , 4].

17. *a.* Calculate the area between the curves $y = x^3$ and $y = x$.

b. Rotate the area in (*a*) about the *y*-axis and give the integral that determines the value. Do not evaluate the integral.

c. Rotate the area in (*a*) about the *x*-axis and give the integral that determines the value. Do not evaluate the integral.