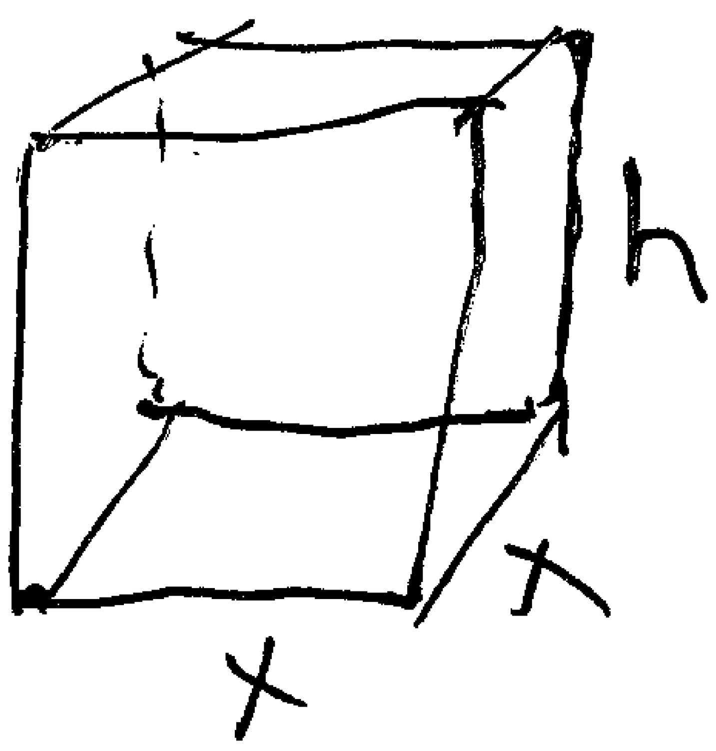
MAT 151 Summer 2009 Test 3 Version 1 Name: Key

For complete credit, show all work.

1. Find the dimensions of a rectangular box with a square base and a closed top that will minimize the cost of materials. The base (bottom) and top cost 20 cents per square inch, and the sides cost 16 cents per square inch. The volume of the box is to be 80 cubic inches.



Volume $= x^2 h = 80 \Rightarrow h = 80x^{-2}$

Min Cost $= 20(2x^2) + 16(4xh)$

$= 40x^2 + 64x(80x^{-2})$

Min $(0, +\infty)$ $f(x) = 40x^2 + 64(80)x^{-1}$

$f'(x) = 80x - 64(80)x^{-2} = 0$

$80x = \frac{64(80)}{x^2}$

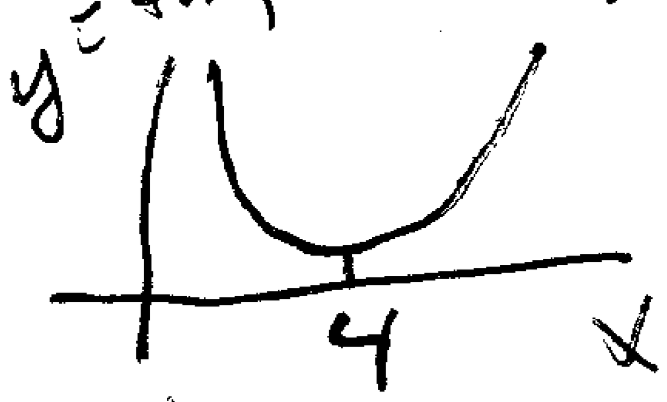
$x^3 = 64$

$x = 4$

$f'(4) = 0$

$f''(x) = 80 + 64(80)x^{-3}$

$f''(4) = 80 + \frac{64(80)}{64} = 160 > 0$



Cost is min when

$\boxed{x = 4 \text{ inches} \quad h = \frac{80}{4^2} = \frac{80}{16} = 5 \text{ inches}}$

2. Find $\frac{dy}{dx}$ by implicit differentiation: $x^7y^4 + 6x^5 = 5x + 3y^7$

$7x^6y^4 + x^7 4y^3\frac{dy}{dx} + 30x^4 = 5 + 21y^6\frac{dy}{dx}$

$(4x^7y^3 - 21y^6)\frac{dy}{dx} = 5 - 7x^6y^4 - 30x^4$

$$\boxed{\frac{dy}{dx} = \frac{5 - 7x^6y^4 - 30x^4}{4x^7y^3 - 21y^6}}$$

3. Calculate the following:

a. $\int \left(12x^{-11} + 3x - e^{4x}\right)dx$

$$\frac{12x^{-10}}{-10} + \frac{3}{2}x^2 - \frac{e^{4x}}{4} + C$$

$$\boxed{-\frac{6}{5}x^{-10} + \frac{3}{2}x^2 - \frac{1}{4}e^{4x} + C}$$

b. $\int_2^4 \left(6x^2 + 2\right)dx = \frac{6x^3}{3} + 2x \Big|_2^4$

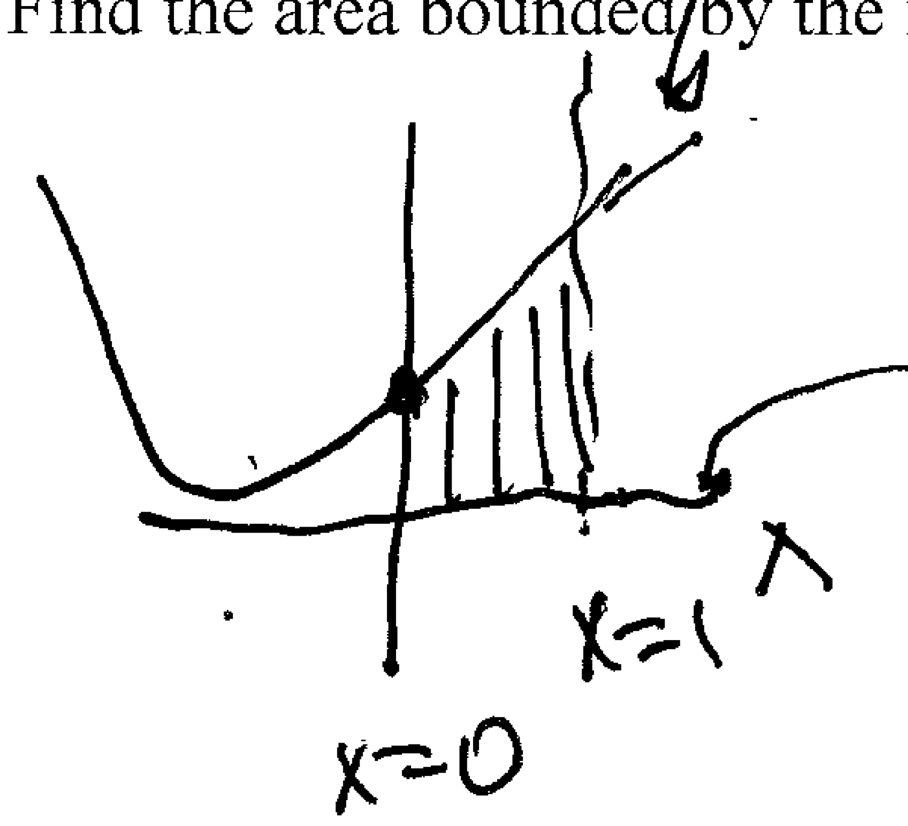
$$= \left(2x^3 + 2x\right)\Big|_2^4$$

$$= \left(2(4^3) + 2(4)\right) - \left(2 \cdot (2^3) + 2(2)\right)$$

$$= 4\left[2 \cdot 4^2 + 2 - (2^2 + 1)\right] = 4\left[34 - 5\right] = 4(29)$$

$$y = 2x^2 + 2x + 5 \qquad = \boxed{116}$$

4. Find the area bounded by the following curves: $y = 2x^2 + 2x + 5$, x = 0, $x = 1$, and $y = 0$.



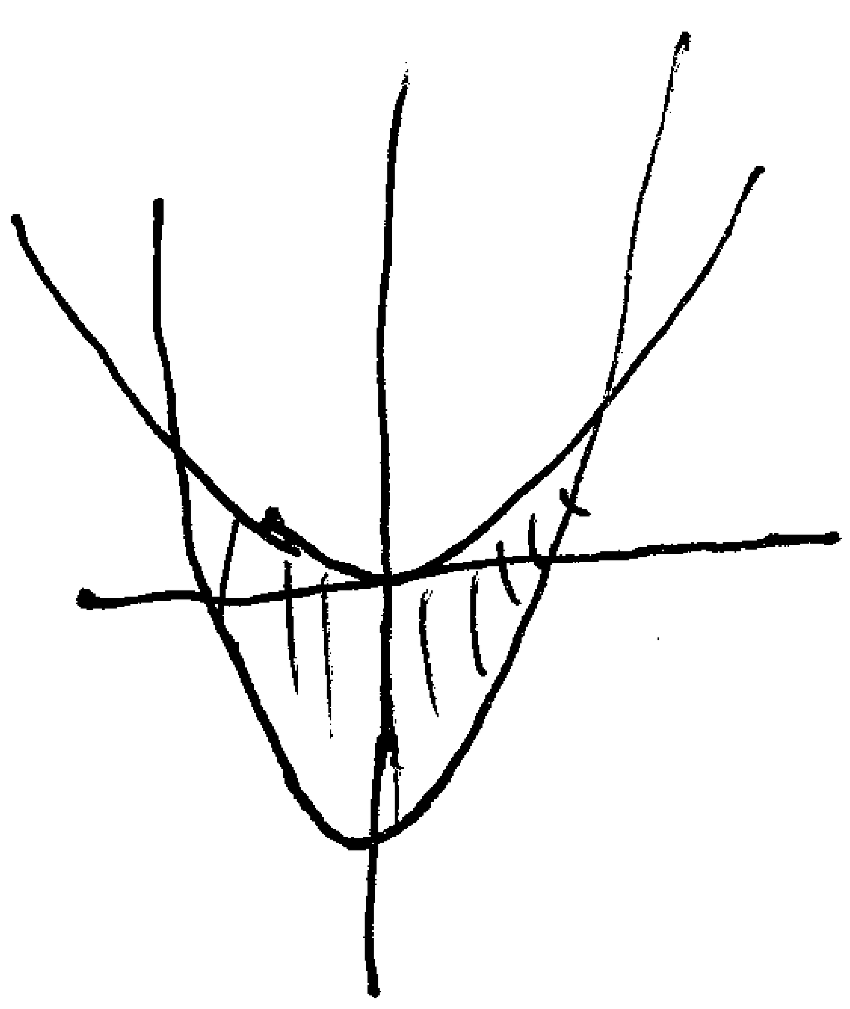
$y = 0$

$$\int_0^1 \left(2x^2 + 2x + 5\right)dx$$

$$\left(\frac{2}{3}x^3 + x^2 + 5x\right)\Big|_0^1$$

$$\boxed{\frac{2}{3} + 1 + 5} = \boxed{6\frac{2}{3}}$$

5. Find the area between the curves y = x^2 and y = $4x^2 - 10$.



$$x^2 = 4x^2 - 10 \rightarrow x = \pm\sqrt{\frac{10}{3}}$$

$$10 = 3x^2$$

$$\frac{10}{3} = x^2$$

$$\int_{-\sqrt{\frac{10}{3}}}^{\sqrt{10/3}} x^2 - (4x^2 - 10)\,dx$$

$$\left(-x^3 + 10x\right)\Big|_{-\sqrt{10/3}}^{\sqrt{10/3}}$$

$$x(-x^2 + 10)\Big|_{-\sqrt{\frac{10}{3}}}^{\sqrt{\frac{10}{3}}}$$

$$= \sqrt{\frac{10}{3}}\left(-\frac{10}{3} + 10\right) \cancel{+ \sqrt{\frac{10}{3}}\left(-\frac{10}{3} + 10\right)}$$

$$= \boxed{2\sqrt{\frac{10}{3}}\left(\frac{2}{3}(10)\right)}$$

6. Calculate the following: $\int \frac{e^{2x}+2}{e^{2x}+4x}\,dx$ $= \int \frac{1}{w} \frac{1}{2}\, dw$

$w = e^{2x} + 4x$

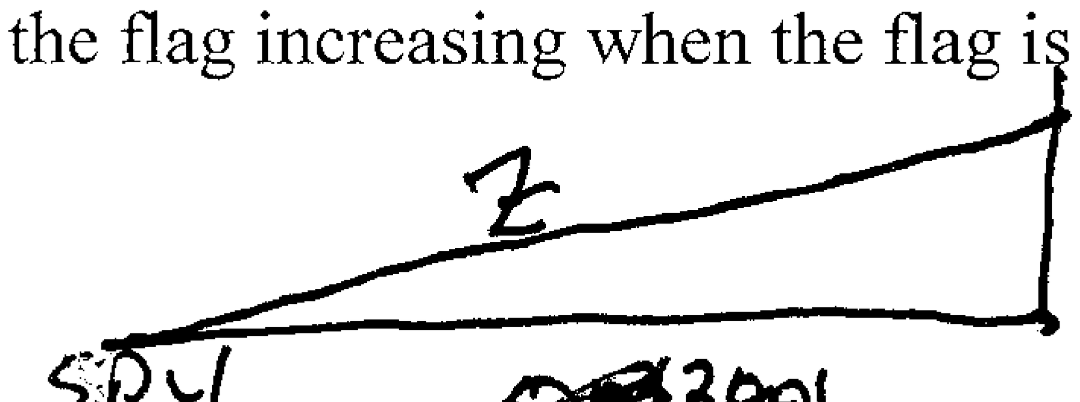
$\frac{dw}{dx} = e^{2x}2 + 4 = 2(e^{2x}+2)$

$\frac{1}{2}dw = (e^{2x}+2)dx$

$= \frac{1}{2}\ln|w| + C$

$= \boxed{\frac{1}{2}\ln|e^{2x}+4x| + C}$

7. A spy lying on the ground is watching a flag being raised at the rate of 2 feet per second on a vertical pole 50 feet tall. The spy is 300 feet from the base of the pole.. How fast is the distance from the spy to the flag increasing when the flag is halfway up the pole?



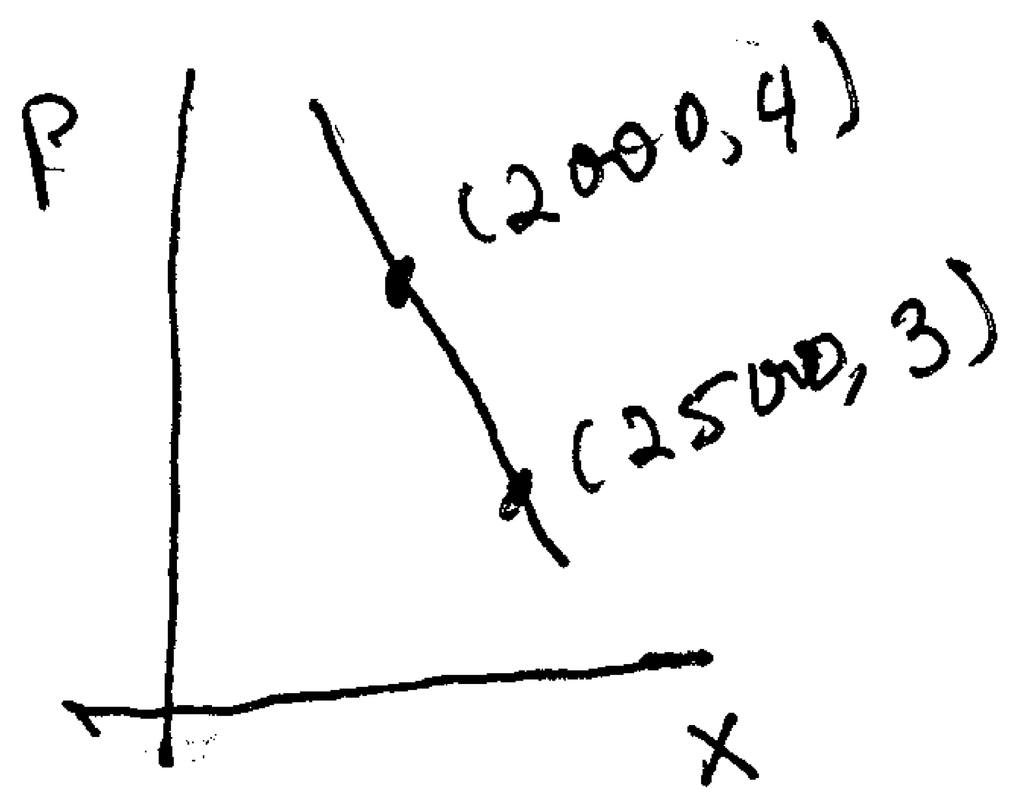
$\frac{d\boxed{z}}{dt} = ?$ when $h = 25$; $\frac{d\boxed{h}}{dt} = 2$ ft/sec.

$z^2 = 300^2 + h^2$

$2z\frac{dz}{dt} = 0 + 2h\frac{dh}{dt}$

$\frac{dz}{dt} = \frac{\cancel{2}h\frac{dh}{dt}}{\cancel{2}z} = \frac{(25)(2)}{\sqrt{300^2+(25)^2}} = \frac{2}{\sqrt{145}}$ ft/second

8. The Varsity can sell 2500 hamburgers per day at a price of \$3 per hamburger and 2000 hamburgers per day at a price of \$4 per day. Assume a linear demand function and that the total cost per day of processing x hamburgers is 6x + 200 dollars per day. How many hamburgers should be produced and sold per day in order to maximize profits. (extra credit)



$m = \frac{4-3}{2000-2500} = \frac{-1}{500}$

$P - 4 = -\frac{1}{500}(x - 2000)$

$P = -\frac{1}{500}x + 8$

Profits = Revenue - Cost

$= xP - (6x + 200)$

$= x(-\frac{1}{500}x + 8) - (6x + 200)$

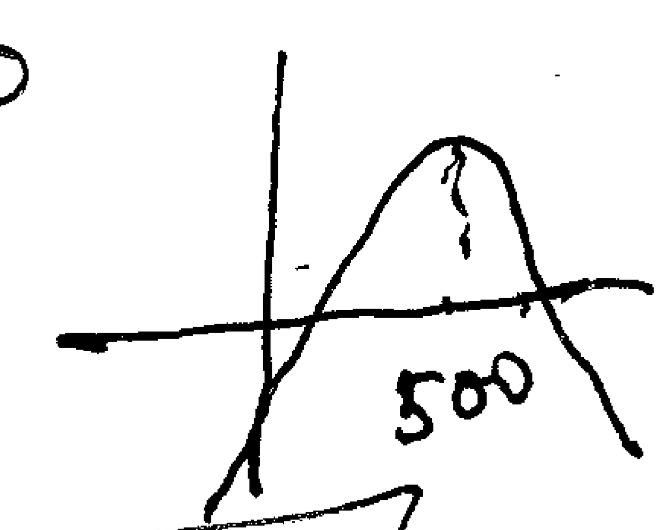
$f(x) = -\frac{1}{500}x^2 + 2x - 200$

$f'(x) = -\frac{1}{250}x + 2 = 0$

$2 = \frac{x}{250}$

$x = 500$

$f''(x) = -\frac{1}{250}$



$\boxed{x = 500}$