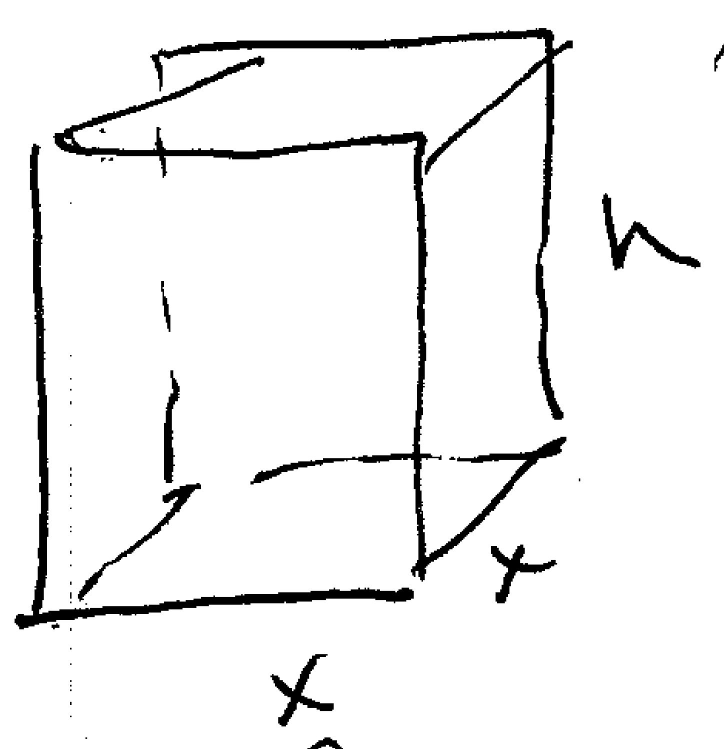
MAT 151 Fall 2014 Test 3 Name: Key

For complete credit, show all work.

1. Find the dimensions of a rectangular box with a square base and a closed top that will minimize the cost of materials. The base (bottom) and top cost 10 cents per square inch, and the sides cost 8 cents per square inch. The volume of the box is to be 200 cubic inches.

20



2

$2\ V = x^2 h = 200 \Rightarrow h = 200x^{-2}$ 2

Min Cost $= 10(2x^2) + 8(4xh)$ 2

$f(x) = 20x^2 + 32(200x^{-1})$ 2 min $(0, \infty)$

$f'(x) = 40x - 32(200)x^{-2} = 0$

$40x = \frac{32(200)}{x^2}$

4 $x^3 = \frac{32(\cancel{200})}{\cancel{40}} = 32(5) = 160$

$x = (160)^{\frac{1}{3}}$

2

$f'(160^{\frac{1}{3}}) = 0$

$x = 160^{\frac{1}{3}} \approx 5.4288$

$2\ f''(x) = 40 + 64(200)x^{-3}$

$f''(160^{\frac{1}{3}}) > 0$

$2\ h = \frac{200}{160^{2/3}} = 6.786$

6

2. Find $\frac{dy}{dx}$ by implicit differentiation: $y^7x^4 + 6x^5 = 5x + 3y^7$

20

$7y^6\frac{dy}{dx}x^4 + y^7 4x^3 + 30x^4 = 5 + 21y^6\frac{dy}{dx}$

2 2 2 2

$5\ (7y^6x^4 - 21y^6)\frac{dy}{dx} = 5 - 4y^7x^3 - 30x^4$

$5\ \frac{dy}{dx} = \frac{5 - 4y^7x^3 - 30x^4}{7y^6x^4 - 21y^6}$

3. Calculate the following:

a. $\int \left(12x^{-10} + 7x - e^{5x}\right)dx$

15

$\frac{12x^{-9}}{-9} + \frac{7x^2}{2} - \frac{e^{5x}}{5} + C$

4 4 4 3

1 2

15 b. $\int_2^4 \left(9x^2 + 2\right)dx$

$3x^3 + 2x \Big|_2^4 = 3(4^3) + 2(4) - (3(2^3) + 2(2))$

4 4

$= 192 + 8 - (28) = 200 - 28$

$= 172$

4. Find the area between the curves y = x^2 and y = 4x^2 – 27.

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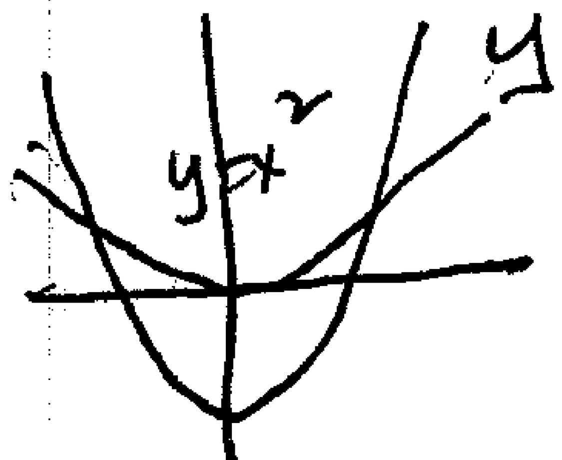


2

$y = 4x^2 - 27 = x^2$

$3x^2 = 27$

$x^2 = 9$

$x = \pm 3$

$\int_{-3}^{3} x^2 - (4x^2 - 27)\,dx$

$= \int_{-3}^{3} (-3x^2 + 27)\,dx$

$= -x^3 + 27x \Big|_{-3}^{3}$

$= -(27) + 27(3) - (-(-27) - 27(3))$

$= -27 + 27(3) - 27 + 27(3) = 27(4)$

$= 108$

5. Two students are walking in opposite directions on the centers of two parallel tracks whose centers are 10 feet apart. One students is walking at a rate of 5 feet per second and the other at 8 feet per second. How fast are they separating 10 seconds after they pass each other?

15

$\frac{dx}{dt} = 5$

$\frac{dy}{dt} = 8$

$\frac{dz}{dt} = ?$ $t = 10$

2 $z^2 = 10^2 + (x+y)^2$

$2z\frac{dz}{dt} = 2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$

$\frac{dz}{dt} = \frac{x+y}{z}\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$

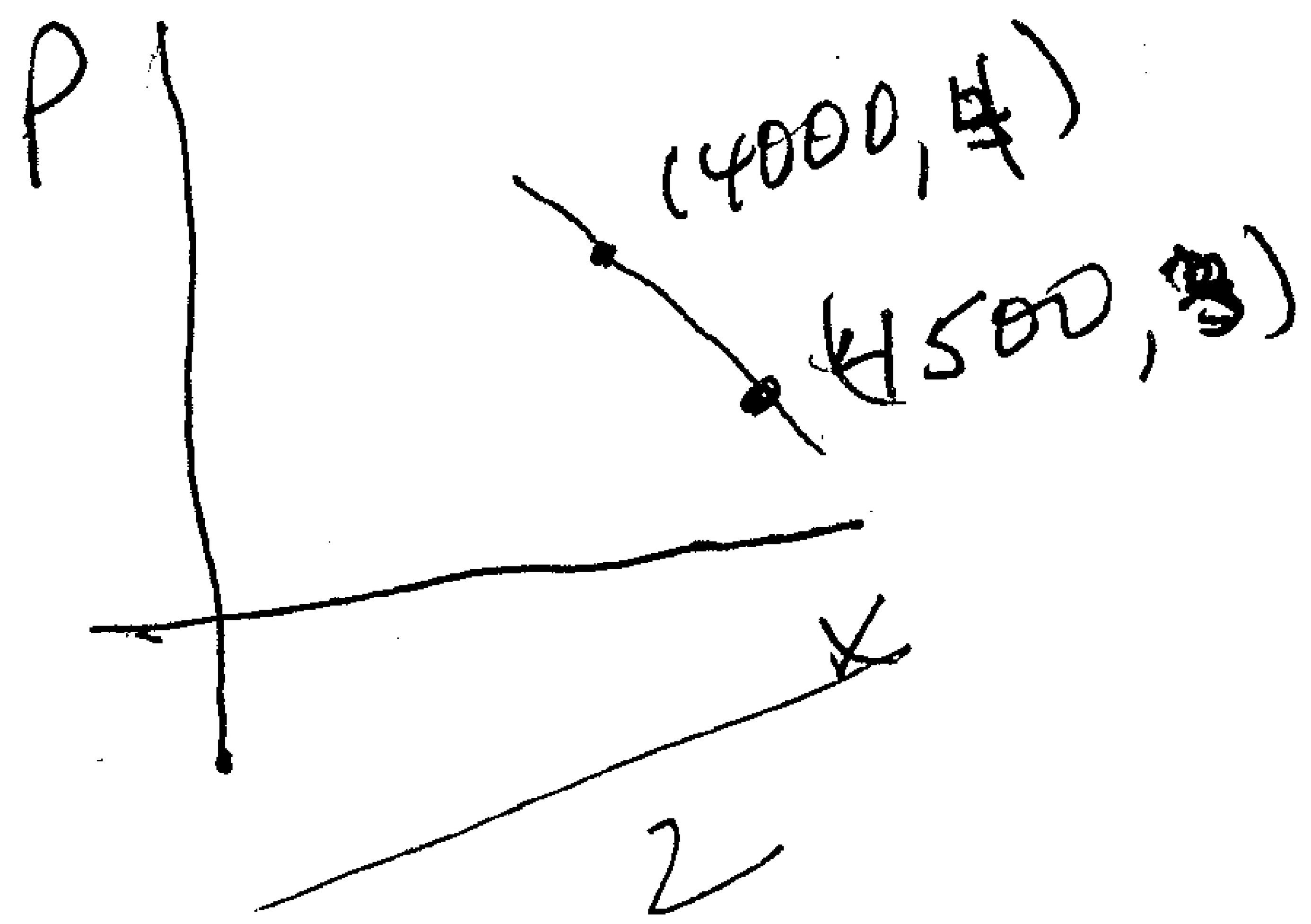
$= \frac{50+80}{\sqrt{10^2 + (130)^2}}(5+8) = \frac{13(130)}{10\sqrt{1+13^2}}$

$= \frac{13}{\sqrt{170}}$

12.96

6. The Varsity can sell 4500 hamburgers per day at a price of $3 per hamburger and 4000 hamburgers per day at a price of $4 per day. Assume a linear demand function and that the total cost per day of processing x hamburgers is 2x + 200 dollars per day. How many hamburgers should be produced and sold per day in order to maximize profits. (extra credit)

15



$m = \frac{4-3}{-500} = \frac{1}{-500}$

$P - 4 = \left(-\frac{1}{500}\right)(x - 4000)$

$P = -\frac{1}{500}x + 8 + 4$

$P = -\frac{1}{500}x + 12$

4

Profits = Revenue − Cost

$= xP - (2x + 200)$ 4

$= -\frac{1}{500}x^2 + 12x - 2x - 200$ 2

$f(x) = -\frac{1}{500}x^2 + 10x - 200$

$f'(x) = -\frac{1}{250}x + 10 = 0$

$10 = \frac{1}{250}x$ 1

$2500 = x$

$f'(2500) = 0$

$\boxed{x = 2500}$ 1

$f''(x) = -\frac{1}{250}$

$f''(2500) < 0$