MAT 151 Fall 2014 Test 2 Name: Key

For complete credit, show all work.

In problems 1-5, find the derivative of the following functions.

1. $f(x) = \dfrac{\ln(3x^8+5)}{5x+7}$

$$f'(x) = \frac{(5x+7)\frac{24x^7}{3x^8+5} - [\ln(3x^8+5)][5]}{(5x+7)^2}$$

2. $g(x) = e^{(2+7x)}(3x^4+7)$

$$g'(x) = e^{2+7x}(7)(3x^4+7) + e^{2+7x}(12x^3+0)$$

3. $h(x) = (12x^4 - 27x^{1/9})^{-5}$

$$h'(x) = (-5)(12x^4 - 27x^{\frac{1}{9}})^{-6}(48x^3 - 3x^{-8/9})$$

4. $k(x) = 3x^6 + 4^{8x}$

$$k'(x) = 36x^5 + 4^{8x}(\ln 4)8$$

5. $m(x) = 8x + \ln(6x+9)$

$$m'(x) = 8 + \frac{6}{6x+9}$$

6. Find the equation of the tangent line to the curve $y = f(x) = 5x^2 - 30x+7$ that has a slope of 1.

$f'(x) = 10x - 30 = 1$

$10x = 31$

$x = 3.1$

$f(3.1) = 5(3.1)^2 - 30(3.1) + 7$

$= 3.1[5(3.1) - 30] + 7$

$= 3.1[5.5 - 30] + 7$

$= (3.1)(-14.5) + 7 = -37.95$

$y - -37.95 = 1(x - 3.1)$

$y = x - 3.1 - 37.95$

$y = x - 41.05$

7. The value in 10 years of \$4000 invested with an interest rate of r compounded monthly is

$FV = 4000(1+r/12)^{120}$ dollars..

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Find the rate of change of FV with respect to r when $i = .06$.

$\frac{dFV}{dr} = 120(4000)\left(1+\frac{r}{12}\right)^{119}\left(\frac{1}{12}\right)$

$= 10(4000)\left(1+\frac{r}{12}\right)^{119}$

$r = .04 \quad = 10(4000)\left(1+\frac{.06}{12}\right)^{119}$

$= 10(4000)(1+.005)^{119}$

$= 72413.00036$

8. For $f(x) = (x-5)^2(6x+5)$,

a. find the intervals where $f(x)$ is increasing and decreasing.

$f'(x) = 2(x-5)(6x+5) + (x-5)^2 6 = 2(x-5)(6x+5+3(x-5))$

$= 2(x-5)(9x-10) = 18(x-5)(x-\frac{10}{9})$

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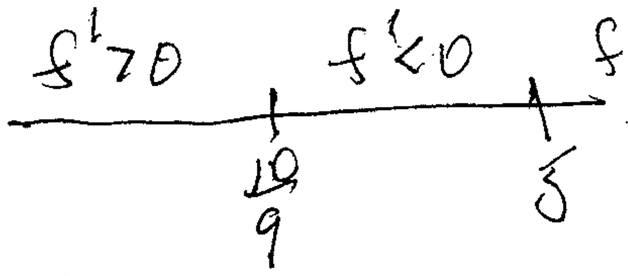

f is inc on $(-\infty, \frac{10}{9}) \cup (5, +\infty)$

f is dec on $(\frac{10}{9}, 5)$

b. find the intervals where $f(x)$ is concave up and concave down.

$f''(x) = 18(x-\frac{10}{9}) + 18(x-5) = 18(2x-\frac{55}{9}) = 36(x-\frac{55}{18})$

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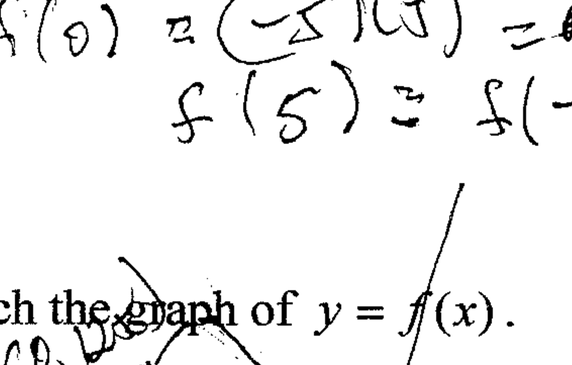


f is concave down on $(-\infty, \frac{55}{18})$

f is concave up on $(\frac{55}{18}, +\infty)$

c. find where $f(x)$ has a relative max and relative min.

f has a rel max at $x = \frac{10}{9}$

5 f has a rel. min at $x = 5$

d. find where $f(x)$ has an inflection point.

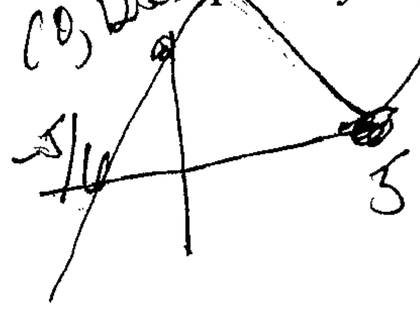
5 f has an inflection point at $x = \frac{55}{18}$

e. find the x and y intercepts of $y = f(x)$.

$f(0) = (-5)(5) = 125$ $(0, 125)$ is the y-intercept.

5 $f(5) = f(-\frac{5}{6}) = 0$ $(5, 0)$ & $(-\frac{5}{6})$ are the x-intercepts

f. sketch the graph of $y = f(x)$.



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g. find the absolute minimum and absolute maximum of f(x) on the interval [2,6].

$= 153$

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$f(2) = (-3)^2(17) = 9(17)$ ← absmax

$f(5) = 0$ ← absmin

$f(6) = 1^2(36+5)$