MAT 151 Fall 2014 Test 1—Version 1 Name ______________________

For complete credit, show all work.

In problems 1-4, calculate the limits.

8 1. $\lim_{x\to5}\frac{x^2+2x-35}{x-5} = \lim_{x\to5}\frac{(x+7)(x-5)}{x-5} = \lim_{x\to5}(x+7) = 5+7 = 12$ 3   3   2

8 2. $\lim_{x\to2}\frac{8x^2+8x}{x+2} = \lim_{x\to2}\frac{8(2^2)+8(2)}{2+2} = \frac{8(2^2+2)}{4} = 2(6) = 12$ 4   4

8 3. $\lim_{x\to6}\frac{x^2-36}{|x-6|}$ $\lim_{x\to6^-}\frac{(x-6)(x+6)}{-(x-6)} = -12$; $\lim_{x\to6^+}\frac{(x-6)(x+6)}{x-6} = 12$ 3   3

$\therefore \lim_{x\to6} f(x)$ does not exist   2

8 4. $\lim_{x\to+\infty}\frac{9+2x}{7x-6} = \frac{2}{7}$ leading coefficients   4   4

10 5. The height h of a small bird flying above the ground is measured by the following equation where h is in feet and x is seconds between 0 and 8:

$$h = 36 - (x-2)^2 = 36 - x^2 + 4x - 4 = 32 - x^2 + 4x$$

Find the vertex of the parabola. What is the meaning of the x-intercept of the graph?

3 $x = \frac{-4}{2(-1)} = 2$, $h = 36$
$(2, 36)$ is the vertex
1 1

$x = 8$, $h = 0$ 3
At 8 seconds the bird lands on the ground. 2

6. Find the horizontal and vertical asymptotes and any x and y intercepts for the following rational function. Draw the graph of the function.

10

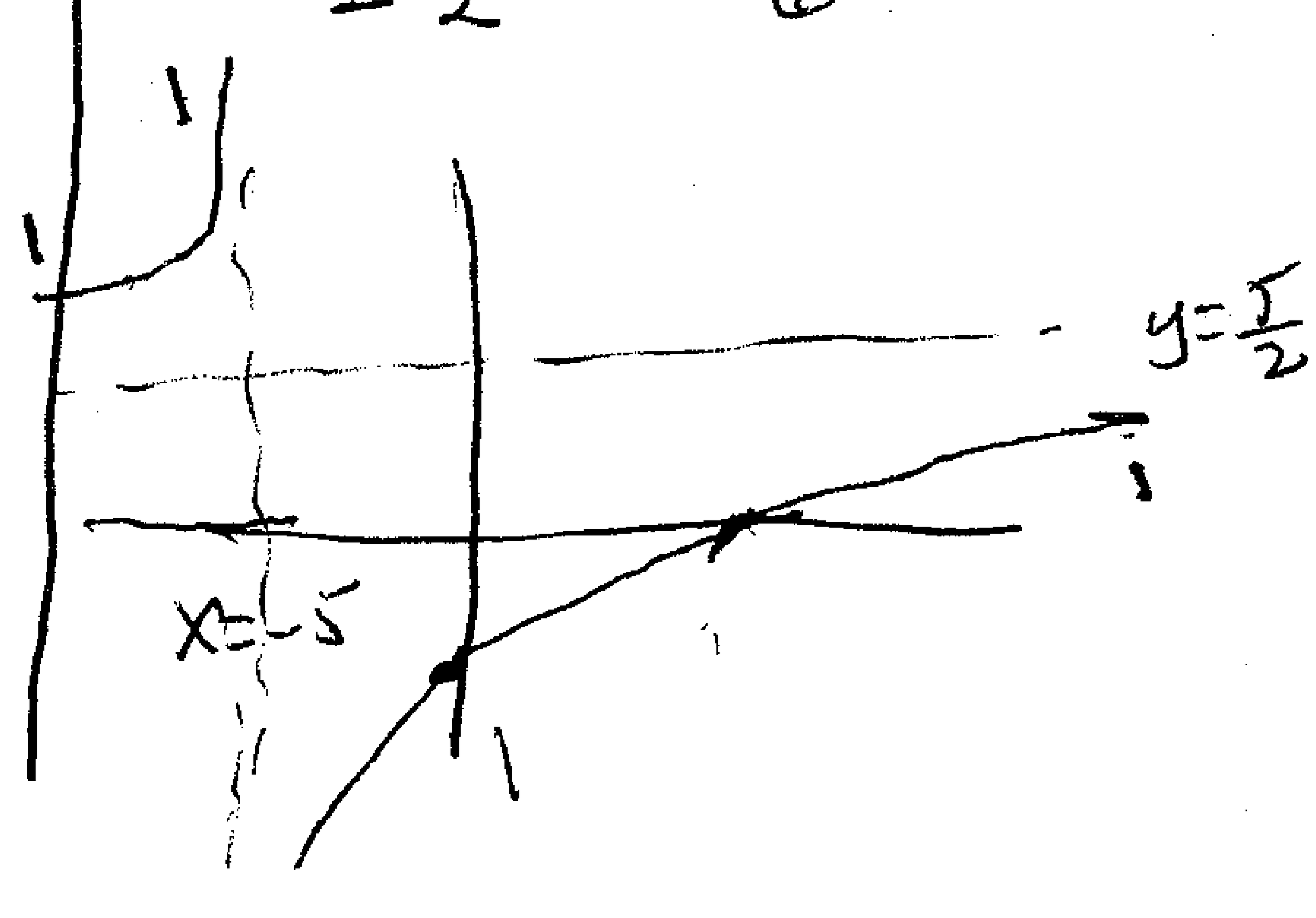
$$f(x) = \frac{5x-30}{2x+10}$$

horizontal
1 $y = \frac{5}{2}$
vertical
1 $x = -5$

$5x - 30 = 0$
$5x = 30$
$x = 6$
1 $(6,0)$ is the x-intercept
$f(0) = \frac{-30}{10} = -3$
1 $(0,-3)$ is the y-intercept

$\frac{5}{2}\frac{(x-6)}{(x+2)}$

$f>0$ $f<0$ $f>0$
-2 6



1

7. Use limits to compute $f'(x)$ where $f(x) = x^2 + 6x + 3$.

10 $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + 6(x+h) + 3 - (x^2 + 6x + 3)}{h}$ 2

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{6x} + 6h + \cancel{3} - (\cancel{x^2} + \cancel{6x} + \cancel{3})}{h}$ 2 $= \lim_{h \to 0} 2x + h + 6$ 2

$= \boxed{2x+6}$ 2

8. Find the equation of the tangent line to the curve $y = x^2 + 6x + 3$ at (2, 19).

10 $f'(2) = 2(2) + 6 = 10$ 2

$y - 19 = 10(x - 2)$ 2 2 2

$y - 19 = 10x - 20$ 2

$\boxed{y = 10x - 1}$

9. An object is thrown from the top of a building that is 525 feet tall. The distance the object is from the ground after t seconds is

$$s(t) = 561 - (4t - 5)^2$$

~~$-2(4t-5)(4)$~~

a. Find the average velocity in the interval [1, 3] seconds.

9 $\frac{s(3) - s(1)}{3 - 1}$ 3 3 3 $= \frac{1^2 - 7^2}{2} = \frac{(1-7)(1+7)}{2} = -3(8) = -24$ ft/second 3

b. Using limits, find the instantaneous velocity at $t = 1$ seconds.

9 $\lim_{h \to 0} \frac{s(1+h) - s(1)}{h}$ 3 $= \lim_{h \to 0} \frac{\cancel{561} - (4(1+h) - 5)^2 - (\cancel{561} - (-1)^2)}{h}$ 2 2

$= \lim_{h \to 0} \frac{1^2 - (4h - 1)^2}{h}$ 2 2 $= \lim_{h \to 0} \frac{1 - 16h^2 + 8h - \cancel{1}}{h}$

$= 4(2) = 8$ ft/second 2

10. Find a so that $f(x) = \begin{cases} 5x + 7 & \text{if} \quad x \le 2 \\ -2x + a & \text{if} \quad x > 2 \end{cases}$ is continuous at x = 2.

10 $f(2) = 17$

$-2(2) + a = 17$

$\boxed{a = 21}$

$-2(4t-5)4$