Test 1 MAT 361 Gurganus Spring 2012 Name: Key Attached

Solve the differential equations in 1-5 and then do 6-8. Show all work for full credit.

1. $2x + 4x^3y^5 + (5x^4y^4 + 3y^2)\frac{dy}{dx} = 0.$

2. $\frac{dy}{dx} + 3x^2y = (x+1)e^{-x^3}.$

3. $\frac{dy}{dx} = y^2x + x.$

4. $\frac{dy}{dx} = \frac{y^2+xy}{x^2}$ (treat as a homogeneous equation)

5. $\frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x^2}$ *(treat as a Bernoulli equation)*

6. Using Euler's method and a stepsize of .01 to estimate y(2.01) if y(2) = 3 and $\frac{dy}{dx} = 5x^2 + \arctan(\frac{3x}{2y}).$

7. If there were no harvesting, an Atlantic fish population grows at a rate directly proportional to the population itself. The annual growth rate is .05 and the initial population is 2,000,000 units. The fishing industry harvests 150,000 units per year. When will the population become extinct?

8. Multiply the differential equation in problem 1 by x and show that the resulting equation is not exact. Find the integrating factor for the resulting equation.

4. $\frac{dy}{dx} = \frac{y^2+xy}{x^2} = \left(\frac{y}{x}\right)^2 + \frac{y}{x}$ 2 $\quad v = \frac{y}{x}$

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$y = vx$

$\frac{dy}{dx} = \frac{dv}{dx}x + v$

$\frac{dv}{dx}x + \cancel{v} = v^2 + \cancel{v}$ 2

$\int \frac{1}{v^2}\,dv = \int \frac{1}{x}\,dx$ 2

3 $-\frac{1}{v} = \ln|x| + c$

3 $v = \frac{1}{-(\ln|x| + c)}$

3 $y = \frac{x}{-(\ln|x| + c)}$

5. $\frac{dy}{dx} - \frac{y}{x} = \frac{1}{x^2}y^2$ $\quad$ 2 $v = y^{1-2} = \frac{1}{y}$

15

2 $\frac{dv}{dx} = \frac{-1}{y^2}\frac{dy}{dx}$

2 $\left\{ \begin{array}{l} -\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{yx} = -\frac{1}{x^2} \\ \frac{dv}{dx} + \frac{1}{x}v = -\frac{1}{x^2} \end{array} \right.$

2 $\mu = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

2 $\frac{d}{dx}(vx) = -\frac{1}{x}$

$vx = -\ln|x| + c$

2

2 $v = \frac{-\ln|x| + c}{x}$

1 $y = \frac{x}{-\ln|x| + c}$

6. $f(x,y) = 5x^2 + \arctan\left(\frac{3x}{2y}\right)$

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$f(2,3) = 5\cdot 4 + \arctan\left(\frac{6}{6}\right)$

$= 20 + \frac{\pi}{4}$

x	y	f(x,y) h
1 2	3 1	$(20 + \frac{\pi}{4})(.01)$ 3
1 2.01	$\boxed{3 + .2 + \frac{\pi}{400}}$ 3	

3.207854

1. $\underbrace{2x + 4x^3 y^5}_{M} + (\underbrace{5x^4y^4 + 3y^2}_{N})\frac{dy}{dx} = 0$

15

3 $\frac{\partial M}{\partial y} = 20x^3y^4$ $\quad \frac{\partial N}{\partial x} = 20x^3y^4$

3 $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ exact is exact

3 $\phi(x,y) = \int M\,dx = \int (2x + 4x^3y^5)\,dx = x^2 + x^4y^5 + g(y)$

3 $x^4 5y^4 + g'(y) = N = 5x^4y^4 + 3y^2 \Rightarrow g'(y) = 3y^2 \Rightarrow g(y) = y^3$

3 $\therefore \boxed{\phi(x,y) = x^2 + x^4y^5 + y^3 = K}$

2. $\frac{dy}{dx} + 3x^2y = (x+1)e^{-x^3}$

15

3 $\mu = e^{\int 3x^2 dx} = e^{x^3}$

4 $\frac{d}{dx}(ye^{x^3}) = x+1$

4 $ye^{x^3} = \frac{1}{2}x^2 + x + c$

4 $\boxed{y = (\frac{1}{2}x^2 + x + c)e^{-x^3}}$

4

3. 3 $\frac{dy}{dx} = y^2x + x = (y^2+1)x$

15

4 $\frac{1}{1+y^2}dy = x\,dx$

4 $\arctan y = \frac{1}{2}x^2 + c$

4 $\boxed{y = \tan(\frac{1}{2}x^2 + c)}$

4

(9) $\frac{dP}{dt} = .05P - 150{,}000$, $P(0) = 2{,}000{,}000$

(9)

$$\frac{dP}{dt} - .05P = -150{,}000$$

$$\mu = e^{-.05t}$$

$$\frac{d}{dt}\left(e^{-.05t}P\right) = -150{,}000\, e^{-.05t}$$

$$e^{-.05t}P = \frac{150{,}000}{.05} e^{-.05t} + C$$

$$P = 3{,}000{,}000 + Ce^{.05t}$$

$$2{,}000{,}000 = P(0) = 3{,}000{,}000 + C$$

$$C = -1{,}000{,}000$$

$$P(t) = 3{,}000{,}000 - 1{,}000{,}000\, e^{.05t}$$

Solve $P(t) = 0$

$$3\,000\,000 - 1000\,000\, e^{.05t} = 0$$

$$e^{-.05t} = \frac{1}{3}$$

$$-.05t = \ln\left(\tfrac{1}{3}\right)$$

$$t = -20 \ln\left(\tfrac{1}{3}\right) = 20 \ln 3 = 21.972$$

8

7 $\underbrace{2x^2 + 4x^4y^5}_{M} + \underbrace{(5x^5y^4 + 3y^3x)}_{N}\frac{dy}{dx} = 0$

1 $\frac{\partial M}{\partial y} = 20x^4y^4 \qquad \frac{\partial N}{\partial x} = 25x^4y^4 + 3y^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ not exact

1 $\mu M + \mu N \frac{dy}{dx}$

$(\mu M)_y = (\mu N)_x$

$\mu_y M + \mu M_y = \mu_x N + \mu N_x$

if $\mu_y = 0$, 1 $\mu M_y = \mu_x N + \mu N_x$

$\mu(M_y - N_x) = \mu_x N$

$\frac{M_y - N_x}{N} = \frac{\mu_x}{\mu} = \frac{d \ln \mu}{dx}$

2 $\frac{20x^4y^4 - (25x^4y^4 + 3y^2)}{5x^5y^4 + 3y^2x} = \frac{-(5x^4y^4 + 3y^2)}{5x^5y^4 + 3y^2x} = -\frac{1}{x} = \frac{d \ln \mu}{dx}$

1 $\ln \mu = -\ln x = \ln \frac{1}{x}$

1 $\boxed{\mu = \frac{1}{x}}$