MAT 162
Spring 2013

Test 4

Name Key

For full credit, show all work.

1. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{10+\cos(\frac{7}{4}\pi n)}{n^2}$ $\leq \sum_{n=1}^{\infty} \frac{11}{n^2} = 11 \sum_{n=1}^{\infty} \frac{1}{n^2}$ which converges ($p=2$ series) 3

13 $\therefore$ By comparison test $\sum_{n=1}^{\infty} \left| \frac{10+\cos(\frac{7}{4}\pi n)}{n^2} \right| \left[\leq \sum_{n=1}^{\infty} \frac{11}{n^2} \right]$ 3

7 $\boxed{\text{Converges absolutely}}$

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{\sqrt{n^5}+6}$ Converges by Alt. Series Test: $f(x) = \frac{x^2+1}{\sqrt{x^5}+6} = \frac{x^2+1}{x^{5/2}+6}$

$f'(x) = \frac{(x^{5/2}+6)(2x) - (x^2+1)\frac{5}{2}x^{3/2}}{(x^{5/2}+6)^2} = \frac{2x^{7/2}+12x-\frac{5}{2}x^{7/2}-\frac{5}{2}x^{3/2}}{(x^{5/2}+6)^2}$

13 3

$= \frac{12x - \frac{1}{2}x^{7/2} - \frac{5}{2}x^{3/2}}{(x^{5/2}+6)^2} < 0$ for all large x

2

① $\therefore f$ decreasing and $f(x) = \frac{1+\frac{1}{x^2}}{x^{\frac{1}{2}}+6x^{-2}} \to 0$ as $x \to +\infty$

② signs alternate

3 But $\sum_{n=1}^{\infty} \frac{n^2+1}{n^{5/2}+6} \geq \sum_{n=1}^{\infty} \frac{n^2}{2n^{5/2}} = \sum_{n=1}^{\infty} \frac{1}{2n^{\frac{1}{2}}}$ diverges $\therefore \sum_{n=1}^{\infty} \frac{n^2+1}{n^{5/2}+6}$ diverges

7 $\boxed{\text{(b) is Conditionally convergent}}$

(c) $\sum_{n=1}^{\infty} (\frac{2n+5}{2n+3})^n$

13 $\lim_{n\to+\infty} \left(\frac{2n+5}{2n+3}\right)^n \neq 0$ since $\left(\frac{2n+5}{2n+3}\right)^n \geq 1$

6

$\therefore$ 7 $\boxed{\text{series diverges}}$

2. Find the radius and interval of convergence for $f(x) = \sum_{n=1}^{\infty} (x-3)^n 5^n n$.

$\lim_{n\to+\infty} \left| \frac{(x-3)^{n+1} 5^{n+1} (n+1)}{(x-3)^n 5^n n} \right| = |x-3| 5 < 1 \Rightarrow |x-3| < \frac{1}{5}$ 3

13 $R = \frac{1}{5}$ $(3-\frac{1}{5}, 3\frac{1}{5})$ 3

at $x = 3+\frac{1}{5}$ $\sum_{n=1}^{\infty} n$ diverges 2

at $x = 3-\frac{1}{5}$ $\sum_{n=1}^{\infty} (-1)^n n$ diverges 2

$\therefore$ $\boxed{\begin{array}{c} R = \frac{1}{5} \quad 1 \\ \text{interval of convergence} \\ \text{is } (2.8, 3.2) \quad 1 \quad 1 \end{array}}$

3. Use a power series to estimate $\int_0^{0.1} \cos(x^8)\,dx$ with an error less than 10^{-20}.

12

$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$; $\cos(x^8) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{16k}}{(2k)!}$; $\int \cos(x^8)\,dx = C + \sum_{k=0}^{\infty} \frac{(-1)^k x^{16k+1}}{(2k)!(16k+1)}$ 3 3 3

$\int_0^{.1} \cos x^8\,dx = \sum_{k=0}^{\infty} \frac{(-1)^k (.1)^{16k+1}}{(2k)!(16k+1)} = \frac{(.1)^1}{1} - \frac{(.1)^{17}}{2!(17)} + \frac{(.1)^{33}}{4!(33)} -$

1 $< 10^{-20}$ 1

$\therefore$ Answer is $\boxed{.1 - \frac{10^{-17}}{34}}$

1

4. Use the fact that the following series is a telescoping series to calculate $\sum_{n=3}^{\infty} \frac{1}{n^2-4}$ exactly.

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$\frac{1}{n^2-4} = \frac{1}{(n-2)(n+2)} = \frac{A}{n-2} + \frac{B}{n+2} \Rightarrow 1 = A(n+2) + B(n-2)$

$n = 2 \quad 1 = A(4) \Rightarrow A = \frac{1}{4}$ 3

$n = -2 \quad 1 = B(-4) \Rightarrow B = -\frac{1}{4}$

$\therefore \frac{1}{n^2-4} = \frac{1}{4}\left[\frac{1}{n-2} - \frac{1}{n+2}\right]$ 3

$\therefore \sum_{n=3}^{\infty} \frac{1}{n^2-4} = \frac{1}{4}\left[\frac{1}{1} - \cancel{\frac{1}{5}}\right] + \frac{1}{4}\left[\frac{1}{2} - \cancel{\frac{1}{6}}\right] + \frac{1}{4}\left[\frac{1}{3} - \cancel{\frac{1}{7}}\right] + \frac{1}{4}\left[\frac{1}{4} - \cancel{\frac{1}{8}}\right] + \frac{1}{4}\left[\cancel{\frac{1}{5}} - \cancel{\frac{1}{9}}\right]$ 3

$+ \frac{1}{4}\left[\cancel{\frac{1}{6}} - \frac{1}{10}\right] + \frac{1}{4}\left[\cancel{\frac{1}{7}} - \frac{1}{11}\right] + \frac{1}{4}\left[\cancel{\frac{1}{8}} - \frac{1}{12}\right] + \frac{1}{4}\left[\cancel{\frac{1}{9}} - \frac{1}{13}\right]$

$+ \cdots$

$= \boxed{\frac{1}{4}\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right]}$ 3 $= \frac{25}{48}$

5. Calculate $\sum_{n=1}^{\infty} \frac{2^{n-1}3^{n+2}}{7^n}$ exactly.

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$= \sum_{n=1}^{\infty} 2^{-1} 3^2 \frac{6^n}{7^n} = \frac{9}{2} \sum_{n=1}^{\infty} \left(\frac{6}{7}\right)^n = \frac{9}{2} \cdot \frac{\frac{6}{7}}{1 - \frac{6}{7}} = \frac{9}{2}(6) = \boxed{27}$

3 3 3 3

recognize
as geometric series 3

6. Use the integral test to determine the number terms in the partial sum for $\sum_{n=1}^{\infty} \frac{1}{n^3}$ that will estimate the infinite series with an error less than .05

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Find n so that $\int_n^{+\infty} \frac{1}{x^3}\,dx < .05$ or

$R_n \lim_{b \to +\infty} \frac{x^{-2}}{-2}\Big|_n^b = \frac{1}{2}\frac{1}{n^2} < .05$

2 2

$\frac{1}{2(.05)} < n^2$ 2

$10 < n^2$

$4 \leq n$ 2

$\boxed{n = 4}$