Test 3 Spring, 2013 Name: Key

MAT 162 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book.

1. For the parametric curve, $x = t^3 - 3t$, $y = t^3 - 3t^2$, $-2 \le t \le 3$:

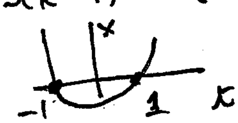
(a) Calculate the following: $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$.

(6) $\frac{dx}{dt} = 3t^2 - 3$; $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t(t-2)}{3(t^2-1)}$; $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{3(t^2-1)(6t-6) - (3t^2-6t \ldots}{[3(t^2-1)]^3}$

$\frac{dy}{dt} = 3t^2 - 6t$

(b) Find when x is increasing and decreasing.

(6) $3(t^2-1) = 3(t-1)(t+1)$

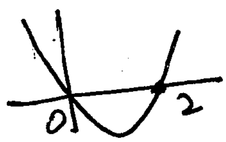


x is inc when $t < -1$ and $t > 1$

x is dec when $-1 < t < 1$

(c) Tell when y is increasing and decreasing.

6



y is inc when $t < 0$ and $t > 2$

y is dec when $0 < t < 2$

(d) Find the xy coordinates where there is a horizontal tangent line.

6

t	x	y
0	0	0
2	2	-2

$\frac{dy}{dt} = 0$ at $t = 0, 2$

(e) Are there any vertical tangent lines? If so, what are the xy coordinates where there is a vertical tangent line?

6

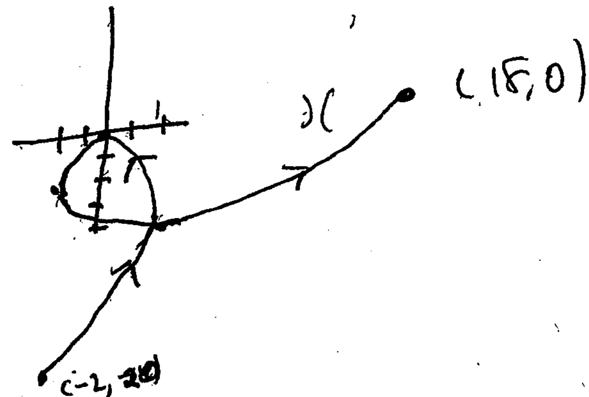
t	x	y
-1	2	-4
1	-2	-2

$\frac{dx}{dt} = 0$ at $t = \pm 1$

(f) Sketch the graph of the parametric curve in the xy plane for $-2 \le t \le 3$.

6

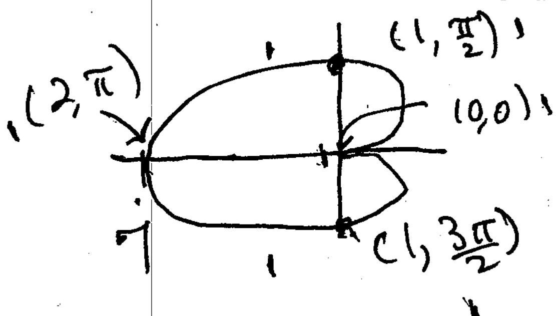
t	x	y	
-2	-2	-20	
-1	2	-4	vertical
0	0	0	horizontal
1	-2	-2	vertical
2	2	-4	horizontal
3	18	0	



(g) Write down, but do not evaluate the integral of the length of the curve.

6 $\int_{-2}^{3} \sqrt{(3t^2-3)^2 + (3t^2-6t)^2}\, dt$

2. (a) Sketch the polar graph of $r = 1 - \cos(\theta)$ from $\theta = 0$ to 2π.. Identify in polar coordinates where the graph crosses the x-axis and y-axis.



(b) Find the area between the curve and the origin in the first quadrant.

8

$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} r^2\, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1-\cos\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 - 2\cos\theta + \cos^2\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(1 - 2\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{3}{2} - 2\cos\theta + \frac{\cos 2\theta}{2}\, d\theta$$

$$= \frac{1}{2}\left(\frac{3}{2}\theta - 2\sin\theta + \frac{\sin 2\theta}{4}\right)\Big|_0^{\frac{\pi}{2}} = \frac{1}{2}\left(\frac{3}{2}\frac{\pi}{2} - 2\right)$$

$$= \frac{1}{2}\left(\frac{3\pi}{4} - 2\right)$$

$$= .17809$$

(c) Write down, but do not evaluate the integral of the length of this curve.

6 $L = \int_0^{2\pi} \sqrt{(1-\cos\theta)^2 + (\sin\theta)^2}\, d\theta$

III. For $16x^2 + 25y^2 - 32x + 100y = 284$,

(a) Identify the conic section.

$16(x^2-2x) + 25(y^2+4y) = 284$

$16(x^2-2x+1) + 25(y^2+4y+4) = 284 + 16 + 100 = 400$

6 $\quad \frac{(x-1)^2}{25} + \frac{(y+2)^2}{16} = 1$

Ellipse

(b) Find the center.

6 $\quad (1, -2)$

(c) Find the vertices.

6 $\quad (1 \pm 5, -2)$ $\quad \left\{ \begin{matrix} (6, -2) \\ (-4, -2) \end{matrix} \right.$

$(1, -2 \pm 4)$ $\quad \left\{ \begin{matrix} (1, 2) \\ (1, -6) \end{matrix} \right.$

(d) Find the foci.

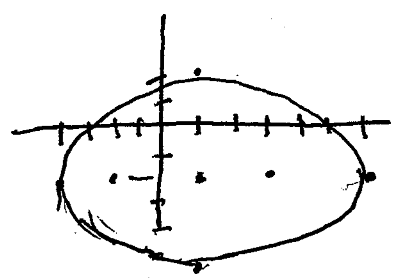
6 $\quad c^2 = 25 - 16$ $\quad (1 \pm 3, -2) = \left\{ \begin{matrix} (4, -2) \\ (-2, -2) \end{matrix} \right.$

$c^2 = 9$

$c = 3$

(e) Sketch the curve in the xy plane.

6



IV. Rewrite the polar equation $r = \frac{2}{1 + 1\sin(\theta)}$ using xy coordinates.

$r + r\sin\theta = 2$

M $\quad r = 2 - r\sin\theta$

$r^2 = (2 - r\sin\theta)^2$

$x^2 + \cancel{y^2} = (2 - y)^2 = 4 - 4y + \cancel{y^2}$

$\boxed{x^2 = 4 - 4y}$