Part 1 Spring, 2013 Name: Key

MAT 162 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator and notes recorded on one side of a single 8.5 by 11 inch paper. Otherwise the test is closed book. When you turn in your test, staple your notes to Part 1.

For 1-4, calculate the following:

(10) 1. $\int x\sec^2(x)dx$ $= \int u\,dv = uv - \int v\,du$

$u = x$ $\quad dv = \sec^2 x\,dx$ $\quad = x\tan x - \int \tan x\,dx$

$du = dx$ $\quad v = \tan x$

$= \boxed{x\tan x + \ln|\cos x| + C}$

(15) 2. $\int \cos^4(x)\sin^5(x)dx$ $= \int \cos^4 x\,(1-\cos^2 x)^2 \sin x\,dx$

$u = \cos x$ $\quad = -\int u^4(1-u^2)^2\,du$

$-du = \sin x\,dx$ $\quad = -\int (u^4 - 2u^6 + u^8)\,du$

$= -\left[\frac{1}{5}\cos^5 x - \frac{2}{7}\cos^7 x + \frac{1}{9}\cos^9 x\right] + C$

(16) 3. $\int \frac{\sqrt{16-x^2}}{x}dx$ $= \int \frac{4\cos\theta\; 4\cos\theta}{4\sin\theta}d\theta$

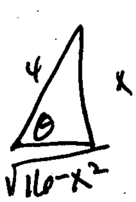
$x = 4\sin\theta$

$= 4\int \frac{\cos^2\theta}{\sin\theta}d\theta = 4\int \frac{1-\sin^2\theta}{\sin\theta}d\theta$

$\sqrt{16-x^2} = 4\cos\theta$

$dx = 4\cos\theta\,d\theta$

$= 4\int(\csc\theta - \sin\theta)d\theta$



$= 4[\ln|\csc\theta - \cot\theta| + \cos\theta]\,d\theta$

$= 4\left[\ln\left|\frac{4}{x} - \frac{\sqrt{16-x^2}}{x}\right| + \frac{\sqrt{16-x^2}}{4}\right] + C$

$= 4\left[\ln\left|\frac{4}{x}\right.\right.$

$= \boxed{\sqrt{16-x^2} + 4\ln\left|\frac{4-\sqrt{16-x^2}}{x}\right| + C}$

(9) 4. $\int \frac{7x+4}{x^2+10x+25}dx$

$\frac{7x+4}{(x+5)^2} = \frac{A}{x+5} + \frac{B}{(x+5)^2} \Rightarrow 7x+4 = A(x+5) + B$

$A = 7$

$4 = 5A+B \Rightarrow B = 4-5A = -31$

$\int \frac{7}{x+5} - \frac{31}{(x+5)^2}dx$

$\boxed{7\ln|x+5| + 31(x+5)^{-1} + C}$

(9) 5. Estimate $\int_4^7 \cos(x^2-1)dx$ using the Trapezoidal Rule with n = 6. Write the sum; you do not have to evaluate the sum.

4.5 5.5 6.5

4 5 6 7

$\boxed{\frac{.5}{2}\left[f(4) + 2f(4.5) + 2f(5) + 2f(5.5) + 2f(6) + 2f(6.5) + f(7)\right] \text{ where } f(x) = \cos(x^2-1)}$

6. Calculate the following; it the integral does not converge, state "does not converge."

(8) a. $\int_1^{+\infty} \frac{1}{x^2+11x+30}dx$

$\frac{1}{(x+6)(x+5)} = \frac{A}{x+6} + \frac{B}{x+5} \Rightarrow 1 = A(x+5) + B(x+6)$

$-1 = A$

$1 = B$

$\int \left(\frac{1}{x+5} - \frac{1}{x+6}\right)dx = \ln|x+5| - \ln|x+6| + C = \ln\left|\frac{x+5}{x+6}\right| + C$

$\int_1^{+\infty} \frac{1}{x^2+11x+30}dx = \lim_{b\to+\infty} \ln\left|\frac{x+5}{x+6}\right| \Big|_1^b = \lim_{b\to+\infty} \ln\left|\frac{b+5}{b+6}\right| - \ln\left(\frac{6}{7}\right)$

$= 0 - \ln\left(\frac{6}{7}\right) = \boxed{\ln\frac{7}{6}}$

(8) b. $\int_0^3 \frac{x}{(9-x^2)\sqrt{9-x^2}}dx = \lim_{b\to3} \int_0^b \frac{x}{(9-x^2)^{3/2}}dx$

$u = 9-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= \lim_{b\to3} \int_9^{9-b^2} u^{-3/2} \cdot -\frac{1}{2}du$

$= \lim_{b\to3} u^{-\frac{1}{2}}\Big|_9^{9-b^2} = \lim_{b\to3} (9-b^2)^{-\frac{1}{2}} - 9^{\frac{1}{2}}$ $\boxed{\text{does not exist}}$

7. Tell why the following converge or diverge:

a. $\int_1^{+\infty} \frac{x^2+1}{(x+1)^3}dx$ compare to $\frac{1}{x}$

$\frac{x^2+1}{(x+1)^3} \geq \frac{x^2}{(2x)^3} = \frac{1}{8}\frac{1}{x}$

$\frac{1}{8}\int_1^{+\infty} \frac{1}{x}dx$ diverges $\Rightarrow \int_1^{+\infty} \frac{x^2+1}{(x+1)^3}dx$ diverges

b. $\int_1^{+\infty} \frac{x^5+1}{2x^7-1}dx$ compare to $\frac{1}{x^2}$

$\frac{x^5+1}{2x^7-1} \leq \frac{x^5+x^5}{2x^7-x^7} = \frac{2x^5}{x^7} = 2\frac{1}{x^2}$

$2\int_1^{+\infty} \frac{1}{x^2}$ converges $\Rightarrow \int_1^{+\infty} \frac{x^5+1}{2x^7-1}dx$ converges

10 8. Calculate $\int \frac{2x+5}{x^2+6x+25}dx = \int \frac{2x+5}{(x+3)^2+4^2}dx = \int \frac{2(x+3)-6+5}{(x+3)^2+4^2}dx$

$$= \int \frac{2(x+3)}{(x+3)^2+4^2}dx - \int \frac{1}{(x+3)^2+4^2}dx$$

$$= \ln((x+3)^2+4^2) - \frac{1}{4}\arctan\left(\frac{x+3}{4}\right) + C$$

$$= \boxed{\ln(x^2+6x+5) - \frac{1}{4}\arctan\left(\frac{x+3}{4}\right)+C}$$

10 9. Write the form of the partial fraction decomposition that you would use to calculate the following integral (you do not have to solve for the constants nor evaluate the integral): $\int \frac{4x+5}{(x^2+12x-28)^2(x^2+2x+17)^3}dx$

$$(x+14)^2(x-2)^2((x+1)^2+4^2)^3$$

$$\boxed{\frac{A_1}{x+14} + \frac{A_2}{(x+14)^2} + \frac{B_1}{x-2} + \frac{B_2}{(x-2)^2} + \frac{C_1x+D_1}{x^2+2x+17} + \frac{C_2x+D_2}{(x^2+2x+17)^2} + \frac{C_3x+D_3}{(x^2+2x+17)^3}}$$