MAT 161 **Test 1** **Name** Key

Fall 2013

Instructions: To receive credit for all answers, show all work clearly in the space provided. You may use graphing calculators. This is designed to be a 50 minute test.

1. Find the indicated limits.

(5 each / 20)

a. $\lim_{x\to 5} \dfrac{x^2-25}{3x^2-8x-35}$

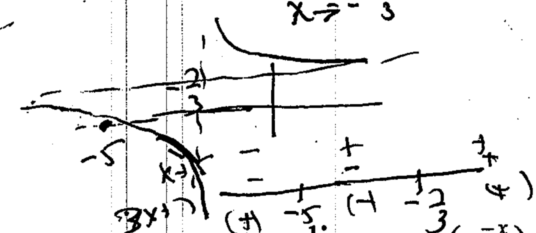
$\lim_{x\to 5} \dfrac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(3x+7)}$ 1 1

$= \dfrac{5+5}{3(5)+7} = \dfrac{10}{22}$ 2

$= \dfrac{5}{11}$ 1

b. $\lim_{x\to -\frac{7}{3}^+} \dfrac{x^2-25}{3x^2-8x-35}$

$\lim_{x\to -\frac{7}{3}^+} \dfrac{x+5}{3x+7} \to \boxed{+\infty}$ 1 1



c. $\lim_{x\to +\infty} \dfrac{x^2-25}{3x^2-8x-35}$

$\frac{1}{3}$ leading coefficients

2 3

d. $\lim_{x\to -\infty} \cos(e^{-x})$ does not exist

3

reason 2

graph no horizontal asymptote

2. Suppose that for all real numbers x, $8x^2 \le f(x) \le x^4+16$, calculate $\lim_{x\to 2} f(x)$.

10

$8(2^2) \le f(x) \le 2^4+16$

$32 \le f(x) \le 32$

$\therefore f(x) = 32$

and $\lim_{x\to 2} 8x^2 = 32 = \lim_{x\to 2} x^4+16$ 3 3

$\therefore$ By Squeeze Law $\lim_{x\to 2} f(x) = 32$ 4

3. Using the precise (δ,ε) definition of limits, prove that $\lim_{x\to 2}(18x-11)=25$

10

Given $\varepsilon>0$ choose $\delta = \frac{\varepsilon}{18}$ 2

Then if $0 \le |x-2| < \delta$ 2

$|x-2| < \frac{\varepsilon}{18}$ 2

$|18x-36| < \varepsilon$ 2

$|18x-11-25| < \varepsilon$ 2

40

4. Suppose that $f(x) = \begin{cases} 4x+10 & \text{if } x \le 3 \\ -x+25 & \text{if } x > 3. \end{cases}$

(5 each)

a. Calculate $\lim_{x\to 3^-} f(x)$.

$4(3)+10=22$

5

b. Calculate $\lim_{x\to 3^+} f(x)$.

$-3+25=22$

5

c. Calculate $f(3)$.

$4(3)+10=22$

5

d. State whether or not $f(x)$ is continuous at $x=3$ and support your answer.

f is cont since $f(3)=22=\lim_{x\to 3} f(x)$

2 1 2

5. Use the Intermediate Value Theorem to find an interval where there is a solution to the equation $4x^3-7=-7x^2+3x$ in the interval.

10 Let $f(x)=4x^3-7+7x^2-3x$ 2

$f(0)=-7$ 2

$f(1)=4-7+7-3=1$ 2

0 is between -7 & 1 2

∴ there is an x between 0 & 1 so that $f(x)=0$. 2

6. The displacement s (in meters) at t seconds of an object moving in a straight line is given by the equation $s=\sqrt{5x+1}$. Find the instantaneous velocity when $x=3$.

10

$\lim_{h\to 0} \frac{s(3+h)-s(3)}{h}$ 1 1 $= \lim_{h\to 0} \frac{\sqrt{5(3+h)+1}-\sqrt{5(3)+1}}{h}$ 2 $= \lim_{h\to 0} \frac{\sqrt{16+5h}-4}{h}$

$= \lim_{h\to 0} \frac{\sqrt{16+5h}-4}{h} \cdot \frac{\sqrt{16+5h}+4}{\sqrt{16+5h}+4}$ 2

$s(3)=4$

-8

$= \lim_{h\to 0} \frac{\cancel{16}+5h-\cancel{16}}{h(\sqrt{16+5h}+4)} = \lim_{h\to 0} \frac{5}{\sqrt{16+5h}+4}$ 2

$= \frac{5}{4+4} = \frac{5}{8}$ 2

7. a. Use the definition of a derivative to find $f'(1)$ where $f(x) = 5x^2 + 2x - 3$.

10

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{5(1+h)^2 + 2(1+h) - 3 - (5+2-3)}{h}$$

$$= \lim_{h \to 0} \frac{5(\cancel{1} + 2h + h^2) + \cancel{2} + 2h - \cancel{3} - \cancel{5} - \cancel{2} + \cancel{3})}{h}$$

$$= \lim_{h \to 0} 5(2+h) + 2 = 12$$

2

-5 -1 $f'(x) = 10x + 2$ but definition not used

-2 if they don't 1x the sq binomial correctly

b. Find an equation of the tangent line to the curve $y = 5x^2 + 2x - 3$ at the point (1 , 4).

10

~~$y - 4 = 4(x - 1)$~~

~~$y - 4 = 4x - 4$~~

~~$\boxed{y = 4x}$~~

$y - 4 = 12(x - 1)$

$y - 4 = 12x - 12$
$\quad +4 \qquad\qquad +4$

$y = 12x - 8$

MAT 161 Test 2 Name Key

Fall 2013

Instructions: To receive credit for all answers, show all work clearly in the space provided.

1. Let $f(x)=x^2e^{-3x}$

   a. Find $f'(x)$.

$f'(x) = 2xe^{-3x} + x^2e^{-3x}(-3)$

$= xe^{-3x}(2-3x)$

   b. Find the equation of the tangent line to the curve $y = f(x)$ at x= 4.

$f(4) = 16e^{-12}$ $\quad 9.83\times10^{-5}$

$f'(4) = 4e^{-12}(2-3(4)) = -40e^{-12} = -2.4577\times10^{-4}$

$y - 16e^{-12} = -40e^{-12}(x-4)$

2. Find the acceleration for a particle moving in a straight line if the position function is

$s(t) = \cos(2t + e^{-t})$.

$v(t) = -\sin(2t+e^{-t})(2-e^{-t})$

$a(t) = -\cos(2t+e^{-t})(2-e^{-t})^2 - \sin(2t+e^{-t})(e^{-t})$

3. Calculate the linearization of $f(x) = x^{\frac{1}{3}}$ at a = 125 Then estimate $(125.15)^{\frac{1}{3}}$ using the linearization.

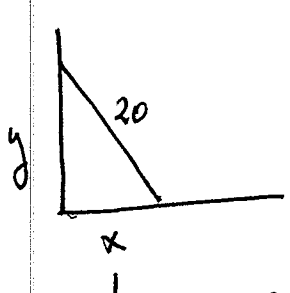
$f(x) = x^{\frac{1}{3}}$ $\qquad f(125) = 125^{\frac{1}{3}} = 5$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$ $\qquad f'(125) = \frac{1}{3}(125)^{-2/3} = \frac{1}{3(25)} = \frac{1}{75}$

$L(x) = 5 + \frac{1}{75}(x-125)$

$L(125.15) = 5 + \frac{1}{75}(.15) = 5 + \frac{1}{100}\left(\frac{1}{5}\right) = 5 + \frac{1}{500} \doteq 5.002$

4. A ladder 20 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft./second, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?



$\frac{dy}{dt} = ?$ when $x = 6$, $\frac{dx}{dt} = 2$ ft/second

$$x^2 + y^2 = 20^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

62897

$$= \frac{-6(2)}{\sqrt{20^2-6^2}} = \frac{-12}{\sqrt{364}} = -.\,\text{[illegible]}\ \text{ft/second}$$

5. The circumference of a circle is measured at 1000 ± 3 cm. Calculate the area of the circle with an estimate for the error using differentials.

$$C = 2\pi r$$

$$\frac{C}{2\pi} = r$$

$$A = \pi r^2 = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{1}{4\pi}C^2$$

$$dA = \frac{1}{2\pi}C\,dC = \frac{1}{2\pi}(1000)3 = \frac{3000}{2\pi}\ \text{cm}^2$$

$$A = \frac{1}{4\pi}1000^2 \pm \frac{3000}{2\pi}$$

$$= 79577.47 \pm 477.46$$

In problems 6 – 10, calculate the derivative of y with respect to x.

6. $y=\dfrac{\sin(3x)+x^2}{x^4+5x^2}$

$$\frac{dy}{dx} = \frac{(x^4+5x^2)(\cos(3x)\,3+2x) - (\sin(3x)+x^2)(4x^3+10x)}{(x^4+5x^2)^2}$$

7. $y=x^3 4^x$

$$\frac{dy}{dx} = 3x^2\, 4^x + x^3\, 4^x \ln 4$$

8. $y=\sin\left(x+\cos^{100}(x)\right)$

$$\frac{dy}{dx} = \cos\left(x+\cos^{100}(x)\right)\left(1+100\cos^{99}(x)(-\sin(x))\right)$$

9. $x^2+5^y y^3=\sin(3x-4y)$

$$2x + 5^y \ln 5 \frac{dy}{dx}\, y^3 + 5^y\, 3y^2 \frac{dy}{dx} = \cos(3x-4y)\left(3-4\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\left(5^y\, 3y^2 + 5^y \ln 5\, y^3 + 4\cos(3x-4y)\right) = 3\cos(3x-4y) - 2x$$

$$\frac{dy}{dx} = \frac{3\cos(3x-4y)-2x}{5^y\, 3y^2 + 5^y \ln 5\, y^3 + 4\cos(3x-4y)}$$

10. $y=x^{\arcsin(x)}$; $\ln y = \arcsin(x)\ln x$

$$\frac{dy}{dx} = y\,\frac{d\ln y}{dx}$$

$$= x^{\arcsin(x)}\left\{\frac{1}{\sqrt{1-x^2}}\ln x + \arcsin(x)\,\frac{1}{x}\right\}$$

MAT 161 Test 3 Name Key

Fall 2013

Instructions: To receive credit for all answers, show all work clearly in the space provided.

1. Find the dimensions of a steel cylinder of surface of 2000 sq. ft, closed-top, that has maximum volume.

Surface Area $= 2\pi r^2 + 2\pi r h = 2000$

2 $\quad h = \dfrac{2000 - 2\pi r^2}{2\pi r}$

2 Max. Volume $= \pi r^2 h = \pi r^2\left[\dfrac{2000 - 2\pi r^2}{2\pi r}\right]$

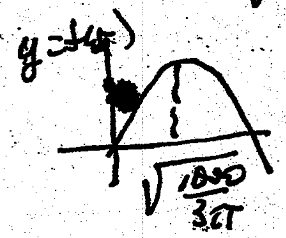
(15) $f(r) = \frac{1}{2}\left[2000r - 2\pi r^3\right] = 1000r - \pi r^3$ on $(0, +\infty)$ 2

4 $f'(r) = 1000 - 3\pi r^2 = 0$

$3\pi r^2 = 1000$

$r = \sqrt{\dfrac{1000}{3\pi}}$

$\rightarrow$ f has a local max at $r = \sqrt{\dfrac{1000}{3\pi}}$ 1



$f''(r) = -6\pi r$

3 $f'\left(\sqrt{\dfrac{1000}{3\pi}}\right) = 0$

$f''\left(\sqrt{\dfrac{1000}{3\pi}}\right) < 0$

f has abs max at $r = \sqrt{\dfrac{1000}{3\pi}} = 10.3$

$h = \dfrac{2000 - 2\pi\left(\frac{1000}{3\pi}\right)}{2\pi\left(\sqrt{\frac{1000}{3\pi}}\right)} = 20.6$ .1

2. Given a demand function of $p(x) = 10000 - 4x$ dollars and a total cost function of $C(x) = 200x$ dollars, determine the production level x that will maximize profits.

Max Profits = Revenue - Cost

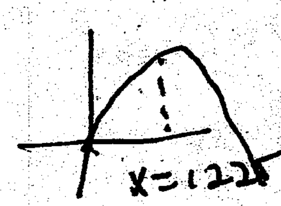
(15) (3) $= xp - 20x$

(2) Max $f(x) = x(10000 - 4x) - 200x = 9800x - 4x^2$ on $[0, +\infty)$

2 $f'(x) = 9800 - 8x = 0$

3 $x = \dfrac{9800}{8} = 1225$

$\rightarrow$ f max when $x = 1225$



$f''(x) = -8$

3 $f'(1225) = 0$

$f''(1225) < 0$

$\boxed{x = 1225}$

2

30

3. Calculate $\lim_{x\to 0} \frac{\sin(5x)-5x}{x^3}$

10

$= \lim_{x\to 0} \frac{\cos(5x)(5)-5}{3x^2}$ 3

$= \lim_{x\to 0} \frac{-\sin(5x)(25)}{6x} = \lim_{x\to 0} \frac{-\cos(5x)\,125}{6} = \frac{-125}{6}$ 3 3 1

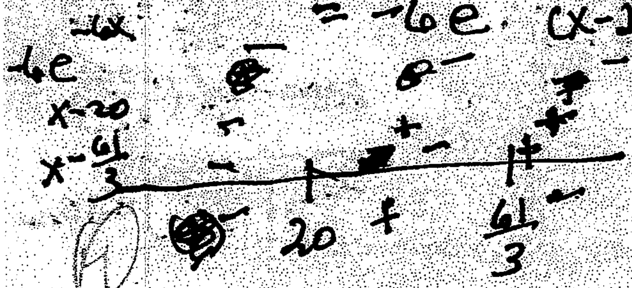
4. Find where f(x) = $e^{-6x}(x-20)^2$ has a relative max.

10

$f'(x) = e^{-6x}(-6)(x-20)^2 + 2e^{-6x}(x-20)$ 3 1

$= 2e^{-6x}(x-20)[-3(x-20)+1] = 2e^{-6x}(x-20)(-3x+61)$

$= -6e^{-6x}(x-20)(x-\frac{61}{3})$



$\boxed{\text{rel. max at } x = \frac{61}{3} \text{ by 2D Test}}$ 1

5. Use an initial guess of 101 and Newton's Method once to estimate the solution to $(x\text{-}100)^2x + 1 = 0$. Tell why a guess of 100 would not be good.

10

$f(x) = (x-100)^2 x + 1$

$f'(x) = 2(x-100)x + (x-100)^2$; $f'(101) = 2(101) + 1^2 = 203$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ 2 2 2

$= 101 - \frac{f(101)}{f'(101)}$

$= 101 - \frac{102}{203} = \boxed{100.4975369}$

$f'(100) = 0$

$\therefore x_{n+1}$ not defined if $x_n = 100$ 1

6. For $f(x) = x^4 - 72x^2$

40

a. Calculate the first and second derivative of *f(x)*.

$f'(x) = 4x^3 - 144x$ 4

$f''(x) = 12x^2 - 144$ 3

7

b. Find the intervals where *f(x)* is increasing and decreasing.

$f'(x) = 4x(x^2-36) = 4x(x-6)(x+6)$

x+6 − + + +
4x − − + +
x−6 − − − +
(−) −6 (+) 0 (−) 6 (+)

3

f is increasing on $(-6,0) \cup (6,+\infty)$ 2

f is decreasing on $(-\infty,-6) \cup (6,+\infty)$ 2

7

c. Find the intervals where *f(x)* is concave up and concave down.

$f''(x) = 12(x^2-12) = 12(x-3\sqrt{2})(x+3\sqrt{2})$ ($2\sqrt{3}$, $2\sqrt{3}$)

$x+3\sqrt{2}$ − + +
$x-3\sqrt{2}$ − − +
+ $-3\sqrt{2}$ ($-2\sqrt{3}$) − $3\sqrt{2}$ ($+2\sqrt{3}$) +

3

f is concave up on $(-\infty,-3\sqrt{2}) \cup (3\sqrt{2},+\infty)$ 2

f is concave down on $(-3\sqrt{2}, 3\sqrt{2})$

7

d. Identify the local maximum, local minimum, and inflection points.

2

f has local min's at $x = \pm 6$ 3

f has local max at $x = 0$ 1

f has inflection points at $x = \pm 3\sqrt{2}$ ($2\sqrt{3}$) 3

7

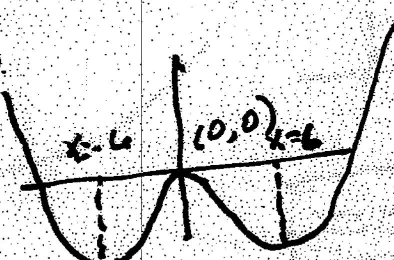
e. Find the *y*-intercepts of *y* = *f(x)*.

$f(0) = 0$ 6

[Did not ask for x-intercepts]

6

f. Sketch the graph of *y* = *f(x)*.



6

40

MAT 161
Fall , 2014

Test 4

Name Key

Show all work for credit purposes.

1. Evaluate the Riemann sum for $f(x) = 4x^2 + 5$ on $2 \le x \le 6$, with four subintervals, taking the sample points to be the right endpoints.

(10)

3 2 3 4 5 6

$\Delta x = \frac{6-2}{4} = 1$ 3

3

$1\,(f(3) + f(4) + f(5) + f(6))$

$41 + 69 + 105 + 149 = 364$ 4

2. Suppose the second derivative of a function is $4x^2 + \sin(x)$ and for the function $f(0) = 10$ and $f(\pi) = 5$. Find the function

(10)

$f''(x) = 4x^2 + \sin(x)$

3 $f'(x) = \frac{4}{3}x^3 - \cos(x) + C_1$

3 $f(x) = \frac{x^4}{3} - \sin(x) + C_1 x + C_2$

2 $10 = f(0) = C_2$

$f(x) = \frac{1}{3}x^4 - \sin x + C_1 x + 10$

2 $5 = f(\pi) = \frac{1}{3}\pi^4 - 0 + C_1 \pi + 10 \Rightarrow C_1 = \frac{-5 - \frac{1}{3}\pi^4}{\pi} = -11.92692499$

$f(x) = \frac{x^4}{3} - \sin(x) + \frac{-5 - \frac{1}{3}\pi^4}{\pi} x + 10$

3. Calculate $\lim_{n\to\infty} \sum_{k=1}^{n} \left(-1 + k\frac{3}{n}\right)^5 \left(\frac{3}{n}\right)$ by evaluating the equivalent integral.

1 $b - a = 3$

1 $a = -1$

1 $b = 2$

1 $f(x) = x^5$

$\int_{-1}^{2} x^5\,dx = \left.\frac{x^6}{6}\right|_{-1}^{2}$

$= \boxed{\frac{2^6}{6} - \frac{1}{6}}$

$= \frac{63}{6} = \frac{21}{2}$

4. Find the area from $x=1$ to $x=5$, between the x-axis and the curve $y=x^2+\frac{3}{x}$.

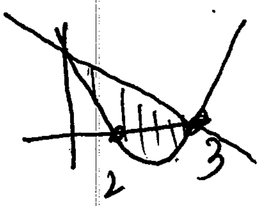
(10)

$$\int_1^5 \left(x^2+\frac{3}{x}\right)dx = \left(\tfrac{1}{3}x^3+3\ln x\right)\Big|_1^5$$

$$= \tfrac{1}{3}5^3+3\ln 5-\tfrac{1}{3} = \cancel{42.97...} = 46.16164707$$

2

5. Calculate the area bounded by the curves $y=(x-2)(x-3)$ and $y=-2x+6$.



$(x-2)(x-3) = -2x+6$

$x^2-5x+6 = -2x+6$

$x^2-3x=0$

$x(x-3)=0$

$x=0 \quad x=3$

2

(11)

$$\int_0^3 2\pi x\left(-2x+6-(x-2)(x-3)\right)dx$$

$$\int_0^3 2\pi x(3x-x^2)\,dx$$

$$\int_0^3 2\pi(3x^2-x^3)\,dx$$

$$2\pi\left(x^3-\tfrac{1}{4}x^4\right)\Big|_0^3$$

$$= \boxed{2\pi\left(3^3-\tfrac{1}{4}3^4\right)}$$

$= 2\pi(27)\frac{1}{4}$

$= \boxed{\frac{27\pi}{2}}$

$= 13.5\pi$

6. Find the volume of the solid formed by rotating the area in question 5 about the Y-axis.

$$\int_0^3 (-2x+6)-(x-2)(x-3)\,dx = \int_0^3 (3x-x^2)\,dx$$

$$= \frac{3}{2}x^2-\frac{x^3}{3}\Big|_0^3$$

$$= 27\left(\tfrac{1}{2}-\tfrac{1}{3}\right)$$

$$= 27\left(\tfrac{1}{6}\right) = 4.5$$

7. Calculate the following.

(20)

a. $\int\left(\frac{5}{\sqrt{1-x^2}}+\cos x\right)dx$

$5\arcsin x + \sin x + C$

3 3 1

(7)

b. $\int x^6 \sin(x^7+1)dx$

(6)

$w = x^7+1$

$\frac{dw}{dx} = 7x^6$

$\frac{1}{7}dw = x^6 dx$

$\int \sin(w)\frac{1}{7}dw$

$\frac{1}{7}(-\cos w)+C$

$-\frac{1}{7}\cos(x^7+1)+C$

c. $\int_{-1}^{2}(3x+4)^5 dx$

(7)

$w = 3x+4$

$\frac{dw}{dx} = 3$

$\frac{1}{3}dw = dx$

$\int_1^{10} w^5 \cdot \frac{1}{3}dw$

$\frac{w^6}{18}\Big|_1^{10}$

$\frac{1}{18}(10^6-1)$

8. Find the derivative of the following function of x: $\int_{-1}^{3x+2}\tan(4t+5)^5 dt$

(10)

$y = \int_{-1}^{w}\tan(4t+5)^5 dt \qquad w = 3x+2$

$\frac{dy}{dx} = \frac{dy}{dw}\frac{dw}{dx} = \tan(4w+5)^5(3)$

$= \tan(4(3x+2)+5)^5(3)$

2 each

9. A particle is moving in a straight line with a velocity of 5t-10 feet per second from t= 3 to t=6 seconds. How far has the particle moved during that 2 second interval?

(10) $s(6)-s(3) = \int_3^6 (5t-10)\,dt = \frac{5}{2}t^2-10t\Big|_3^6$

3 3 2 3

$= \frac{5}{2}(36)-10(6) - \left[\frac{5}{2}3^2-10(3)\right]$

$= 30-(-7.5) = 37.5$