MAT 161
Spring, 2012

Test 4 Version 1

Name Key

Show all work for credit purposes.

1. Evaluate the Riemann sum for $f(x)=8x-2$ on $2\le x\le 10$, with four subintervals, taking the sample points to be the midpoints.

10

3

2 4 6 8 10

$\Delta x = \frac{10-2}{4} = 2$ 3

$2(f(3)+f(5)+f(7)+f(9)) = 2(22+38+54+70) = 2(184)$

$= 368$

1 2 1

2. Find the derivative of $y=\int_0^{x^2}\cos(t^2+2)\,dt$

10

1. $y = \int_0^u \cos(t^2+2)\,dt$ $u = x^2$ 1

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \cos(u^2+2)\,2x = \cos(x^4+2)\,2x$

2 2 1 2 1 1

3. Calculate $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+k\frac{4}{n}\right)^2\left(\frac{4}{n}\right)$ by evaluating the equivalent integral.

10

1 $b-a=4$

1 $a=3$

1 $b=7$

1 $\int_3^7 x^2\,dx = \frac{1}{3}x^3\Big|_3^7$ 1

1 2

$= \frac{1}{3}7^3 - \frac{1}{3}3^3 = 105\frac{1}{3} = \frac{316}{3}$

1 1

4. Find the area from $x=1$ to $x=4$, between the x-axis and the curve $y=6x^2+\cos(x)$.

10

1 $\int_1^4 (6x^2+\cos(x))\,dx = (2x^3+\sin(x))\Big|_1^4 = 2(4^3)+\sin(4) - 2 - \sin(1)$ 1

1 2 2 2

$= 2(63)+\sin 4 - \sin 1$

≈ 127.65

5. Find the average value of $f(x) = x + \sin(5x)$ on the interval [2, 10].

10

$$\frac{1}{10-2}\int_2^{10} x+\sin(5x)\,dx = \frac{1}{8}\left(\frac{1}{2}x^2 - \frac{1}{5}\cos(5x)\right)\Big|_2^{10}$$

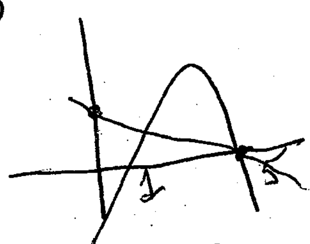
$$= \frac{1}{8}\left(\frac{100}{2} - \frac{1}{5}\cos 50 - \left(2 - \frac{1}{5}\cos 10\right)\right)$$

$$= \frac{1}{8}\left(50 - \frac{1}{5}\cos 50 - 2 + \frac{1}{5}\cos 10\right)$$

$$= \frac{1}{8}\left(48 - \frac{1}{5}\cos 50 + \frac{1}{5}\cos 10\right) \approx 5.95$$

6. Calculate the area bounded by the curves $y = (1-x)(x-5)$ and $3y + 5x = 25$.

10



$3(1-x)(x-5) = 25-5x$

$3(1-x)(x-5) = 5(5-x)$

$3(1-x)(x-5) + (x-5)5 = 0$

$(x-5)[3(1-x)+5] = 0$

$(x-5)[-3x+8] = 0$

$(-3)(x-5)(x-\frac{8}{3}) = 0$

$$\int_{8/3}^{5} (1-x)(x-5) - \left(\tfrac{1}{3}\right)(25-5x) \approx 2.117$$

$$\int_{8/3}^{5} -x^2 + x + 5x - 5 - \frac{25}{3} + \frac{5}{3}x \,dx$$

$$\int_{8/3}^{5} -x^2 + \left(7\tfrac{2}{3}\right)x - \frac{40}{3}\,dx$$

$$= -\frac{1}{3}x^3 + \left(7\tfrac{2}{3}\right)\frac{x^2}{2} - \frac{40}{3}x \Big|_{8/3}^{5}$$

$\approx 12.5 - (-14.617) \approx 2.117$

7. a. Rotate the area in number 6 about the X-axis and write the integral that is the resulting volume. You do not have to evaluate the integral.

5

$$\int_{8/3}^{5} \pi\left[(1-x)^2(x-5)^2 - \left(\tfrac{1}{3}(25-5x)\right)^2\right]dx$$

b. Rotate the area in number 6 about the Y-axia and write the integral that is the resulting volume. You do not have to evaluate the integral.

5

$$\int_{8/3}^{5} 2\pi x\left[(1-x)(x-5) - \tfrac{1}{3}(25-5x)\right]dx$$

8. Calculate the following.

7 7

a. $\int\left(\frac{17x}{1+x^2}+\sec x\tan x\right)dx$ b. $\int x^2 cox(x^3+5)dx$ c. $\int_{-1}^{5}(7x-35)^4\,dx$

6 $17\frac{1}{2}\ln(1+x^2)+\sec x+C$

$u=x^3+5$

$\frac{du}{dx}=3x^2$

$\frac{1}{3}du=x^2\,dx$

$\int \cos u\,\frac{1}{3}du$

$(\sin u)\frac{1}{3}+C$

$\boxed{\frac{1}{3}\sin(x^3+5)+C}$ 2

$u=7x-35$

$\frac{du}{dx}=7$

$\frac{1}{7}du=dx$

$\int_{-42}^{0}u^4\,\frac{1}{7}\,du$

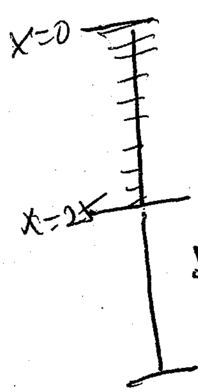
$\frac{1}{7}\frac{1}{5}u^5\Big|_{-42}^{0}$

$\boxed{\frac{1}{7}\frac{1}{5}\left[42^5\right]}$

3734038.2

9. A 50 foot cable weighing a total of 250 lbs is hanging over the edge of a tall bridge. Find the amount of work done to reel in 25 feet of the cable to the edge of the bridge.

10



$5\int_0^{25} x\,dx$

$\frac{1}{2}(250)=125$ lbs.

$\frac{5\,dx}{1}+125(25)$

$\frac{5}{2}x^2\Big|_0^{25}+125(25)$

$\frac{5}{2}(25)^2+125(25)=25^2\left(\frac{15}{2}\right)$

$=25[5(12.5)+125]$

$=25[187.5]=$ [illegible] ft-lbs.