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MAT 161 **Test 3** **Name** Key

201Spring 2012

Instructions: To receive credit for all answers, show all work clearly in the space provided.

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1. Find the dimensions of a steel cylinder with an exterior surface of 1000 sq. ft, open-top, that has maximum volume.

$S = \pi r^2 + 2\pi r h = 1000$

$h = \frac{1000 - \pi r^2}{2\pi r}$

Max: Volume $= \pi r^2 h = \pi r^2 \left(\frac{1000 - \pi r^2}{2\pi r}\right) = \frac{1}{2}(1000r - \pi r^3)$

$f(r) = \frac{1}{2}[1000r - \pi r^3]$

$f'(r) = \frac{1}{2}[1000 - 3\pi r^2] = 0$

$3\pi r^2 = 1000$

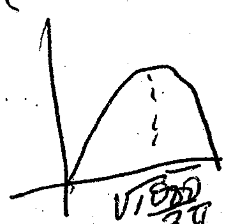
$r^2 = \frac{1000}{3\pi}$

$r = \pm\sqrt{\frac{1000}{3\pi}}$

$= 10.300645$

$f''(r) = \frac{1}{2}[-6\pi r] < 0$

f max when $r = \sqrt{\frac{1000}{3\pi}}$



$h = \frac{1000 - \pi\left(\frac{1000}{3\pi}\right)}{2\pi\sqrt{\frac{1000}{3\pi}}}$

$\frac{1}{\sqrt{3\pi}}\sqrt{1000} \quad \frac{2}{3}(1000)\frac{\sqrt{3\pi}}{2\pi\sqrt{1000}} = 10.300645$

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2. Given a demand function of $p(x) = (2+x)e^{-x}$ thousand dollars, determine the production level x that will maximize revenue.

$f(x) = x(2+x)e^{-x} = (2x + x^2)e^{-x}$ max $f(x)$ on $[0, +\infty)$

$f'(x) = (2+2x)e^{-x} - (2x + x^2)e^{-x}$

$= (2 - x^2)e^{-x}$

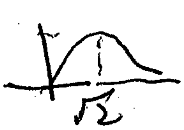
$f'(\sqrt{2}) = 0$

$f''(x) = -2xe^{-x} - (2 - x^2)e^{-x}$

$= (x^2 - 2x - 2)e^{-x}$

$f''(\sqrt{2}) = (2 - 2\sqrt{2} - 2)e^{-\sqrt{2}} = -2\sqrt{2}e^{-\sqrt{2}} < 0$

f max when $x = \sqrt{2} = 1.41421$



3. Calculate $\lim_{x\to 0} \dfrac{\sin(2x)-2x}{\sin(x^3)}$ $= \lim_{x\to 0} \dfrac{\cos(2x)2-2}{\cos(x^3)3x^2}$ 2

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$= \lim_{x\to 0} \dfrac{-\sin(2x)4}{-\sin(x^3)9x^4+\cos(x^3)6x}$ 2

$= \lim_{x\to 0} \dfrac{-\cos(2x)8}{-\cos(x^3)27x^7-\sin(x^3)36x^3-\sin(x^3)18x^3+6\cos(x^3)}$ 2 2

$= \boxed{\frac{-8}{6}} = \boxed{-\frac{4}{3}}$ 2

4. Find f(x) if f "(x) = x + cos(x) , f '(0) = 2 and f(0) = 3.

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$f'(x) = \frac{1}{2}x^2 + \sin(x) + K$ 3

$2 = f'(0) = K$ 1

$f'(x) = \frac{1}{2}x^2 + \sin(x) + 2$ 1

$f(x) = \frac{1}{6}x^3 - \cos(x) + 2x + K$ 3

$3 = -1 + K$

$4 = K$ 1

$\boxed{f(x) = \frac{1}{6}x^3 - \cos(x) + 2x + 4}$ 1

5. Use an initial guess of 1 and Newton's Method once to estimate the solution to x^4- 3x- 3=0.

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$x - \dfrac{f(x)}{f'(x)} = x - \dfrac{x^4-3x-3}{4x^3-3}$ 3

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$= 1 - \dfrac{1-3-3}{4-3}$ 3

$= 1 - (-5)$

$= \boxed{6}$ 1

6. For $f(x) = x^4 - 200x^2$

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a. Calculate the first and second derivative of *f(x)*.

(3) $f'(x) = 4x^3 - 400x$

6 (3) $f''(x) = 12x^2 - 400$

b. Find the intervals where *f(x)* is increasing and decreasing.

$4x(x^2-100) = 4x(x-10)(x+10)$

8 zeros 2, signs 2

$- \quad -10 \quad + \quad 0 \quad - \quad 10 \quad +$

f is decreasing on $(-\infty, -10) \cup (0, 10)$ 2

f is increasing on $(-10, 0) \cup (10, +\infty)$ 2

c. Find the intervals where *f(x)* is concave up and concave down.

$f''(x) = 4(3x^2 - 100) = 4(\sqrt{3}x - 10)(\sqrt{3}x + 10) = \cancel{3}\,12 (x + \frac{10}{\sqrt{3}})(x - \frac{10}{\sqrt{3}})$

8 zeros 2, signs 2

$+ \quad -\frac{10}{\sqrt{3}} \quad - \quad 0 \quad \frac{10}{\sqrt{3}} \quad +$

f is concave up on $(-\infty, \frac{-10}{\sqrt{3}}) \cup (\frac{10}{\sqrt{3}}, +\infty)$ 2

f is concave down on $(\frac{-10}{\sqrt{3}}, \frac{10}{\sqrt{3}})$ 2

d. Identify the local maximum, local minimum, and inflection points.

6 2 f has inflection points at $x = \pm\frac{10}{\sqrt{3}}$

2 min at $x = \pm 10$ $\quad f(\pm 10) = -10{,}000$

2 max at $x = 0$

e. Find the x- and *y*-intercepts of *y = f(x)*.

$x^2(x^2 - 200) = x^2(x - 10\sqrt{2})(x + 10\sqrt{2})$

6 (4) x intercepts at $0, \pm 10\sqrt{2}$

(2) $f(0) = 0$ is the y intercept.

f. Sketch the graph of *y = f(x)*.

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