MAT 161 **Test 2** Name Key

Spring 2012 **Version 1**

Instructions: To receive credit for all answers, show all work clearly in the space provided.

1. Let $f(x) = 5x^2 - 4\sqrt{10x+15} + 22x$

   a. Find $f'(x)$.

   $$f'(x) = 10x - 4\cdot\tfrac{1}{2}(10x+15)^{-\frac{1}{2}}(10) + 22$$

   $$= 10x - \frac{20}{\sqrt{10x+15}} + 22$$

   b. Find the equation of the tangent line to the curve $y = f(x)$ at (1, 7).

   2

   $$f'(1) = 10 - \frac{20}{\sqrt{25}} + 22 = 32 - 4 = 28$$

   $$y - 7 = 28(x-1)$$

   $$y - 7 = 28x - 28$$

   $$y = 28x - 28 + 7$$

   $$\boxed{y = 28x - 21}$$

2. Find the second derivative of $f(x) = \tan(2 + \cos(5x))$.

   10

   $$f'(x) = \sec^2(2+\cos(5x))(0 - \sin(5x)5) = -5\sec^2(2+\cos(5x))\sin 5x$$

   $$f''(x) = -5\left[2\sec(2+\cos(5x))\sec(2+\cos(5x))\tan(2+\cos(5x))(0-\sin(5x))5)\sin(5x) + \sec^2(2+\cos(5x))\cos(5x)5\right]$$

3. Calculate the linearization of $f(x) = x^{\frac{1}{4}}$ at $a = 10000$ Then estimate $(10000.3)^{\frac{1}{4}}$ using the linearization.

   10

   $$f(x) = x^{\frac{1}{4}} \qquad f'(x) = \tfrac{1}{4}x^{-3/4}$$

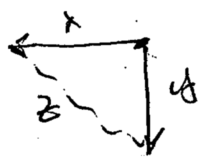
   $$f(10000) = 10000^{\frac{1}{4}} = 10 \qquad f'(10000) = \tfrac{1}{4}\cdot\tfrac{1}{1000}$$

   $$L(x) = f(10000) + f'(10000)(x - 10000)$$

   $$= 10 + \frac{1}{4000}(.3) = 10 + \frac{3}{40000} = 10.000075$$

4. Two hikers leave their camp at noon; one walks west at 3 mph and the other walks south at 4 mph. At what rate are they separating 30 minutes later? Assume the ground is flat.

10



$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = 4$ 1 $\frac{dz}{dt} = ?$ when $t = \frac{1}{2}$ 1

1

2 $z^2 = x^2 + y^2$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$ 1 1

$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$ 1

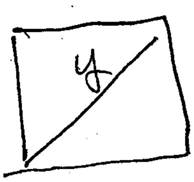
$= \frac{\frac{1}{2}3(3) + \frac{1}{2}4(4)}{\frac{1}{2}\sqrt{3^2+4^2}}$

1 $= \frac{9+16}{5} = (5)$

$= 5$ mph

5. The diagonal of a square is measured at 10 ± .01 cm. Calculate the area of the aquare with an estimate for the error. Hint: Show that the area of a square is $0.5y^2$ where y is the length of the diagonal.

10



$y^2 = x^2 + x^2$

$\frac{y^2}{2} = x^2 = A$

$A = \frac{1}{2}y^2$ 2

3 $A = \frac{1}{2}(10^2) = 50$

error $= dA = \frac{1}{2}(2y)\,dy = y\,dy = 10(.01) = .1$ 2

$A \pm dA$

$\boxed{50 \pm .1 \text{ cm}^2}$

In problems 6 – 10, calculate the derivative of y with respect to x.

6. $y = \dfrac{3x + e^{4x}}{\sin x}$ $= (3x + e^{4x})(\sin x)^{-1}$

10 $\dfrac{dy}{dx} = \dfrac{(\sin x)(3 + e^{4x} 4) - (3x + e^{4x})(\cos x)}{(\sin x)^2}$

or $\dfrac{dy}{dx} = (3 + e^{4x}(4))(\sin x)^{-1} + (3x + e^{4x})(-1)(\sin x)^{-2}\cos x$

7. $y = (\ln(\tan x)) + (\ln x)^2$

10 $\dfrac{dy}{dx} = \dfrac{1}{\tan x} \sec^2 x + 2(\ln x)\dfrac{1}{x}$

2 2 2 2 1 1

2

8. $y = \cos(1 + \cosh x)$

10 $\dfrac{dy}{dx} = -\sin(1 + \cosh x)(0 + \sinh x)$

2 1 1 1 2 1 2

9. $11y^5 - 8yx^3 + x^2 = 3y - x$

10 $11(5y^4)\dfrac{dy}{dx} - 8\dfrac{dy}{dx}x^3 - 8y(3x^2) + 2x = 3\dfrac{dy}{dx} - 1$

$(55y^4 - 8x^3 - 3)\dfrac{dy}{dx} = 24yx^2 - 2x - 1$

$\boxed{\dfrac{dy}{dx} = \dfrac{24yx^2 - 2x - 1}{55y^4 - 8x^3 - 3}}$ 2

10. $y = (x + \sin(x))^{\tan(2x+4)}$

10 $\ln y = \tan(2x+4)\ln(x + \sin(x))$

$\dfrac{dy}{dx} = y\dfrac{d \ln y}{dx} = (x + \sin x)^{\tan(2x+4)}\left\{\sec^2(2x+4)\,2\ln(x + \sin(x)) + \tan(2x+4)\dfrac{1+\cos x}{x + \sin x}\right\}$

-1 -1 -1 -1

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1. Let $f(x) = 6x^2 - 4\sqrt{10x+15} + 21x$

   a. Find $f'(x)$.

5 $f'(x) = 6(2x) - 4(\tfrac{1}{2})(10x+15)^{-\frac{1}{2}}(10) + 21$

$= 12x - \dfrac{20}{\sqrt{10x+15}} + 21$ ✓

   b. Find the equation of the tangent line to the curve $y = f(x)$ at (1, 7).

5 $2\ f'(1) = 12 - \dfrac{20}{\sqrt{25}} + 21 = 33 - 4 = 29$

$y - 7 = 29(x-1)$

$y - 7 = 29x - 29$

$y = 29x - 29 + 7$

$y = 29x - 22$

2. Find the second derivative of $f(x) = \cos(2 + \tan(5x))$.

10 $f'(x) = -\sin(2+\tan(5x))(0 + \sec^2(5x)(5))$

$= -5\sin(2+\tan(5x))\sec^2(5x)$

$f''(x) = -5[\cos(2+\tan(5x))\sec^2(5x)5\ \sec^2(5x)$

$+ \sin(2+\tan(5x))\ 2\sec(5x)\sec(5x)\tan(5x)\ 5]$

$= -25\sec^2 5x[\sec^2(5x)\cos(2+\tan(5x)) + 2\tan(5x)\sin(2+\tan(5x))]$

3. Calculate the linearization of $f(x) = x^{\frac{1}{4}}$ at $a = 625$ Then estimate $(625.3)^{\frac{1}{4}}$ using the linearization.

10 $f(x) = x^{\frac{1}{4}}$ $\qquad f'(x) = \frac{1}{4}x^{-3/4}$

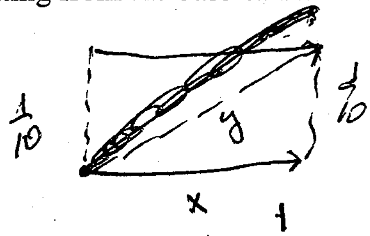
$f(625) = 625^{\frac{1}{4}} = 5$ $\qquad f'(625) = \frac{1}{4}\cdot\frac{1}{5^3} = \frac{1}{4}\cdot\frac{1}{125} = \frac{1}{500}$

$L(x) = f(625) + f'(625)(x - 625)$

$= 5 + \frac{1}{500}(x - 625)$

$L(625.3) = 5 + \frac{1}{500}(.3) = 5 + \frac{.3}{500} = 5.0006$

4. A plane flying 200 mph at an altitude of 0.1 mile flies directly over a tower. At what rate is the plane separating from the base of the tower one minute later.?



1

$\frac{dy}{dt}$ when $t = \frac{1}{60}$

$\frac{dx}{dt} = 200$ mph 1

$x = \frac{1}{60}(200)$

$y = \sqrt{(\frac{1}{10})^2 + (\frac{1}{60})^2(200^2)}$

$y^2 = (\frac{1}{10})^2 + x^2$ 2

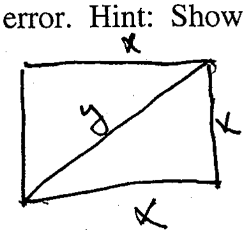
$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$ 2

$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{\frac{200}{60}}{\sqrt{(\frac{1}{10})^2 + (\frac{200}{60})^2}}$ 200 mph $= \frac{200}{\sqrt{(\frac{60}{200})^2(\frac{1}{10})^2 + 1}}$ mph

1 $= \frac{200}{\sqrt{1 + (.03)^2}}$ mph

199.91 mph $= \frac{200}{\sqrt{1 + .0009}} = \frac{200}{\sqrt{1.0009}}$ mph

5. The area of a square is measured at 50 ± .01 cm^2. Calculate the length of the diagonal with an estimate for the error. Hint: Show that the area of a square is $0.5y^2$ where y is the length of the diagonal.



$y^2 = x^2 + x^2$

$\frac{y^2}{2} = x^2 = A$

$A = \frac{1}{2}y^2$ 2

$y = (2A)^{\frac{1}{2}}$ 1

1 $\boxed{y = (2(50))^{\frac{1}{2}} = 100^{\frac{1}{2}} = 10 \text{ cm}}$

$dy = \frac{1}{2}(2A)^{-\frac{1}{2}}(2)\,dA = (2A)^{-\frac{1}{2}}dA = \frac{1}{(2(50))^{\frac{1}{2}}}\,.01$

1 1 1 1

$= \frac{1}{10}(.01) = .001$ 1

1 $\boxed{\text{error} = .001 \text{ cm}}$

$\boxed{10 \pm .001 \text{ cm}}$