MAT 161 Test 1 Name Key

Spring 2012

Instructions: To receive credit for all answers, show all work clearly in the space provided.

1. Find the indicated limits.

$\frac{0}{0}$ L'Hospital's Rule

a. $\lim_{x\to 2} \frac{2x^2-11x-21}{x^2-49}$

$= \frac{2(2^2)-11(2)-21}{2^2-49}$

b. $\lim_{x\to 7} \frac{2x^2-11x-21}{x^2-49}$

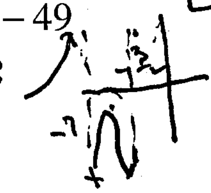
$\lim_{x\to 7} \frac{\cancel{(x-7)}(2x+3)}{\cancel{(x-7)}(x+7)}$

$\frac{2(7)+3}{7+7} = \frac{17}{14}$

$\lim_{x\to 7} \frac{4x-11}{2x} = \frac{4(7)-11}{2(7)} = \frac{17}{14}$

c. $\lim_{x\to -7^+} \frac{2x^2-11x-21}{x^2-49} = \boxed{-\infty}$

$\lim_{x\to -7^+} \frac{2x+3}{x+7}$



$2(x+\frac{3}{2})$ − − +

$x+7$ − + +

+ -7 − $-\frac{3}{2}$ +

d. $\lim_{x\to +\infty} \frac{2x^2-11x-21}{x^2-49}$

$= \frac{2}{1} = 2$

$\lim_{x\to +\infty} \frac{4x-11}{2x}$

$\lim_{x\to +\infty} \frac{4}{2}$

$= 2$

2. Suppose that for all real numbers x between 0 and $\pi/2$, $\frac{4\cos(x)-1}{4-4\sin(x)} \le f(x) \le 1+\sin(x)$.

a. Show that f(x) is continuous at x= $\pi/3$.

$\lim_{x\to \frac{\pi}{3}} 1+\sin(x) = 1+\sin(\frac{\pi}{3}) = 1+\frac{\sqrt{3}}{2}$

$\lim_{x\to \frac{\pi}{3}} \frac{4\cos(x)-1}{4-4\sin(x)} = \frac{4(\frac{1}{2})-1}{4-4\frac{\sqrt{3}}{2}} = \frac{1}{4(1-\frac{\sqrt{3}}{2})} \cdot \frac{(1+\frac{\sqrt{3}}{2})}{(1+\frac{\sqrt{3}}{2})} = \frac{1+\frac{\sqrt{3}}{2}}{4(1-\frac{3}{4})} = 1+\frac{\sqrt{3}}{2}$

$f(\frac{\pi}{3}) = 1+\frac{\sqrt{3}}{2}$ and $\lim_{x\to \frac{\pi}{3}} f(x) = 1+\frac{\sqrt{3}}{2}$ $\therefore$ f is cont at $x=\frac{\pi}{3}$

b. Is f(x) continous at x = $\pi/6$? Why or why not?

$1+\sin(\frac{\pi}{6}) = 1+\frac{1}{2} = \frac{3}{2}$

$\frac{4\cos(\frac{\pi}{6})-1}{4-4\sin(\frac{\pi}{6})} = \frac{4\frac{\sqrt{3}}{2}-1}{4-4(\frac{1}{2})} = \frac{2\sqrt{3}-1}{2} = \sqrt{3}-\frac{1}{2} < \frac{3}{2}$

$\sqrt{3} < 2$

$\boxed{\sqrt{3}-\frac{1}{2} \le f(\frac{\pi}{6}) \le \frac{3}{2}}$

Not necessarily

3. Using the precise (δ, ε) definition of limits, prove that $\lim_{x \to 3} (5x-4) = 11$

Given $\epsilon > 0$ choose $\delta = \frac{\epsilon}{5}$

If $0 < |x-3| < \delta$, then $|x-3| < \frac{\epsilon}{5}$

$|5x-15| < \epsilon$

$|5x-4-11| < \epsilon$

$|f(x) - L| < \epsilon$

4. Use the Intermediate Value Theorem to show that there is a solution to the equation $9 = (x-2)^2 + 7x$ in the interval (1,2). Consider $f(x) = (x-2)^2 + 7x - 9$

f(x) is continuous

$f(1) = 1 + 7 - 9 = -1$

$f(2) = 7(2) - 9 = 5$

0 is a number between -1 & 5

∴ By IVT, there is a number z between 1 & 2 so that $f(z) = 0$

$(z-2)^2 + 7z - 9 = 0$

$(z-2)^2 + 7z = 9$

5. Let $f(x) = \dfrac{3+4x}{1+5x}$. Find the equation of the tangent line to the curve y = f(x) at (2, 1).

$f'(x) = \dfrac{(1+5x)(4) - (3+4x)5}{(1+5x)^2}$

$f'(2) = \dfrac{11(4) - (11)5}{11^2} = -\dfrac{1}{11}$

$\boxed{y - 1 = -\frac{1}{11}(x-2)}$

at the end of the semester

$f'(2) = \lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$

$= \lim_{h \to 0} \dfrac{\frac{3+4(2+h)}{1+5(2+h)} - 1}{h}$

$= \lim_{h \to 0} \dfrac{3+4(2+h) - [1+5(2+h)]}{(1+5(2+h)) \cdot h}$

$= \lim_{h \to 0} \dfrac{\cancel{11} + 4h - (\cancel{11} + 5h)}{(1+5(2+h)) \cdot h}$

$= \lim_{h \to 0} \dfrac{-1}{1+5(2+h)} = -\dfrac{1}{11}$

6. Suppose a stone is thrown vertically upward with an initial velocity of 64 ft/s from a bridge 80 feet above a river. By Newton's Laws of Motion, the position of the stone measured as the height above the river after t seconds is $s(t) = -16t^2 + 64t + 80$ where $s = 0$ is the level of the river.

   a. Find s'(t) .

   $s'(t) = -16(2t) + 64 + 0$

   $= -32t + 64$

   b. Find the average velocity of the stone from t = 1 to t = 2 seconds.

   $\frac{s(2)-s(1)}{2-1} = \frac{-16(2^2)+64(2)+\cancel{80} - (-16+64+\cancel{80})}{1}$

   $-16(3) + 64 = -48 + 64 = 16$ ft/sec.

   c. Find the instantaneous velocity when t = 1 second.

   $s'(1) = -32 + 64 = 32$ ft/sec.

   d. Find the instantaneous velocity when the stone hits the river.

   $-16t^2 + 64t + 80 = 0$

   $t^2 - 4t - 5 = 0$

   $(t-5)(t+1) = 0$

   $t = 5$

   $s'(5) = -32(5) + 64$

   $= -160 + 64$

   $= -96$ ft/sec.