Page 168 − 23, 25, 3ab, 33 Homework Set II

(23) Suppose $2x-1 \le f(x) \le x^2$ for $0 < x < 3$.

$\lim_{x\to 1} 2x-1 = 2-1 = 1$ and $\lim_{x\to 1} x^2 = 1^2 = 1$

Since $\lim_{x\to 1} 2x-1 = \lim_{x\to 1} x^2 = 1$, by the squeeze law $\lim_{x\to 1} f(x) = 1$

(25) Given $\epsilon > 0$, choose $\delta = \frac{\epsilon}{5}$

Then if $0 < |x-2| < \delta$

$|x-2| < \frac{\epsilon}{5}$

$|5x-10| < \epsilon$

$|10-5x| < \epsilon$

$|14-5x-4| < \epsilon$.

(30) $g(x) = \begin{cases} 2x-x^2 & \text{if } 0 \le x \le 2 \\ 2-x & \text{if } 2 < x \le 3 \\ x-4 & \text{if } 3 < x < 4 \\ \pi & \text{if } x \ge 4 \end{cases}$

(a) $\lim_{x\to 2^-} 2x-x^2 = 4-4 = 0$ $g(2) = 0$ $\lim_{x\to 2^+} 2-x = 2-2 = 0$

$\therefore$ g is continuous at 2

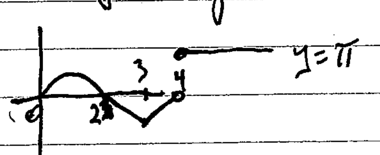
$\lim_{x\to 3^-} 2-x = 2-3 = -1$ $g(3) = -1$ $\lim_{x\to 3^+} (x-4) = 3-4 = -1$

$\therefore$ g is continuous at 3

$\lim_{x\to 4} x-4 = 0$ $g(4) = \pi$ $\lim_{x\to 4^+} \pi = \pi$

$\therefore$ g is right continuous at 4

(b)



(33) Let $g(x) = x^5 - x^3 + 3x - 5$; g is cont since a polynomial

$g(1) = 1-1+3-5 = -2$ $g(2) = 2^5 - 2^3 + 6 - 5 = 25$

0 is between $g(1)$ & $g(2)$

$\therefore$ there is a z between 1 & 2 so that $g(z) = 0$.

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34 ab, 45 ab

(36) $f(x) = \frac{2}{1-3x}$; $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{2}{1-3(x+h)} - \frac{2}{1-3x}}{h}$

$= \lim_{h\to 0} \frac{\cancel{2} - \cancel{6x} - \cancel{2} + 6(x+h)}{h(1-3(x+h))(1-3x)}$

$= \lim_{h\to 0} \frac{6}{(1-3(x+h))(1-3x)} = \frac{6}{(1-3x)^2}$

$f(0) = \frac{2}{1} = 2$; $f'(0) = \frac{6}{1^2} = 6$

$y-2 = 6(x-0) \Rightarrow \boxed{y = 6x+2}$

$f(-1) = \frac{2}{1-3(-1)} = \frac{1}{2}$; $f'(-1) = \frac{6}{(1+3)^2} = \frac{6}{16} = \frac{3}{8}$

$y - \frac{1}{2} = \frac{3}{8}(x - -1)$ or $\boxed{y = \frac{3}{8}x + \frac{7}{8}}$

(37) $s(t) = 1 + 2t + \frac{1}{4}t^2$

(a) For (a) you are calculating $\frac{s(t)-s(1)}{t-1}$ for various values of t

$\frac{s(t)-s(1)}{t-1} = \frac{\cancel{1} + 2t + \frac{1}{4}t^2 - (\cancel{1} + 2(1) + \frac{1}{4}1^2)}{t-1}$

$= \frac{2(t-1) + \frac{1}{4}(t-1)(t+1)}{t-1} = 2 + \frac{1}{4}(t+1)$

(i) $t=3$ $\quad 2 + \frac{1}{4}(3+1) = \boxed{3}$

(ii) $t=2$ $\quad 2 + \frac{1}{4}(2+1) = \boxed{2.75}$

(iii) $t=1.5$ $\quad 2 + \frac{1}{4}(1.5+1) = \boxed{2\frac{5}{8}}$

(iv) $t=1.1$ $\quad 2 + \frac{1}{4}(1.1+1) = \boxed{2.525}$

(b) $s'(1) = \lim_{t\to 1} \frac{s(t)-s(1)}{t-1} = \lim_{t\to 1} 2 + \frac{1}{4}(t+1)$

$= 2 + \frac{1}{2}$

$= \boxed{2.5}$

39 (a) $f(x) = x^3 - 2x$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^3 - 2(2+h) - (2^3 - 2(2))}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2^3} + 3 \cdot 2^2 h + 3(2)h^2 + h^3 - \cancel{4} - 2h - \cancel{2^3} + \cancel{4}}{h}$$

$$= \lim_{h\to 0} 12 + 6h + h^2 - 2 = \boxed{10}$$

(b) $y - 4 = 10(x-2)$

$y = 10x - 20 + 4$

$\boxed{y = 10x - 16}$

45 (a) $f(x) = \sqrt{3-5x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{3-(5(x+h))} - \sqrt{3-5x}}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sqrt{3-5x-5h} - \sqrt{3-5x}}{h}\right) \cdot \left(\frac{\sqrt{3-5x-5h} + \sqrt{3-5x}}{\sqrt{3-5x-5h} + \sqrt{3-5x}}\right)$$

$$= \lim_{h\to 0} \frac{\cancel{3} - \cancel{5x} - 5\cancel{h} - (\cancel{3} - \cancel{5x})}{\cancel{h}} \cdot \frac{1}{\sqrt{3-5x-5h} + \sqrt{3-5x}}$$

$$= \lim_{h\to 0} \frac{-5}{\sqrt{3-5x-5h} + \sqrt{3-5x}} = \boxed{\frac{-5}{2\sqrt{3-5x}}}$$

(b) domain of f is $3 - 5x \geq 0$

$3 \geq 5x$

$\boxed{.6 \geq x}$

domain of f' is $3 - 5x > 0$

$\boxed{.6 > x}$