Group 7

14 $\boxed{\int \frac{x^3}{\sqrt{1+x^2}}\,dx}$

$\int \frac{x^2 x}{\sqrt{1+x^2}}\,dx$ $\qquad u = 1+x^2 \qquad x = \sqrt{u-1}$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{(\sqrt{u-1})^2}{\sqrt{u}}\,du$

$\frac{1}{2}\int \frac{u-1}{\sqrt{u}}\,du$ ✓

$\frac{1}{2}\int \frac{u}{\sqrt{u}}\,du - \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

$\frac{1}{2}\int (u)(u^{-1/2})\,du - \frac{1}{2}\int u^{-1/2}\,du$

$\frac{1}{2}\int u^{1/2}\,du - \frac{1}{2}\int u^{-1/2}\,du$ ✓

$\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)(u^{3/2}) - \left(\frac{1}{2}\right)(2)(u^{1/2}) + C$

$\left(\frac{2}{6}\right)(u^{3/2}) - (u^{1/2}) + C$

$\left(\frac{2}{6}\right)(1+x^2)^{3/2} - (1+x^2)^{1/2} + C$

$\frac{1}{3}(1+x^2)^{3/2} - \sqrt{1+x^2} + C$ ✓

fine

KRG

Group 7

18 $\int_1^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$

$t^{1/2}$

$\frac{1}{2}t^{-1/2}$

$u = \sqrt{t}$

$du = \frac{1}{2}t^{-1/2}\,dt$

$= \frac{1}{2\sqrt{t}}\,dt$

$2\int e^{u}\,du$

$2e^{u}$

$\left[2e^{\sqrt{t}}\right]_1^4$

$\left[2e^{\sqrt{4}}\right] - 2e$

$2e^2 - 2e + \cancel{C}$

C not necessary this is a definite integral KRM

7.5 Assignment

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7/2/2012

25) $\int \frac{3x^2-2}{x^2-2x-8}\,dx$

$$\begin{array}{r} 3 \\ x^2-2x-8 \,\overline{)\, 3x^2+0x-2} \\ \underline{3x^2+6x+24} \\ 6x+22 \end{array}$$

$\frac{3x^2-2}{x^2-2x-8} = 3 + \frac{6x-22}{x^2-2x-8}$

$\frac{6x-22}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2} \Rightarrow 6x+22 = A(x+2)+B(x-4)$

$x=-2$

$-4 \quad 10 = -6B$

$B = -5/3$

$x=4$

$4 \quad 46 = 6A$

$A = \frac{46}{6} = \frac{23}{3}$ ✓

correct here?

$\int \frac{3x^2-2}{x^2-2x-8}\,dx = \int \left(3 + \frac{23/3}{x-4} - \frac{5/3}{x+2}\right)dx$

$= \boxed{3x + 23/3 \ln|x-4| - 5/3 \ln|x+2| + C}$ ✓ fine KRC

26) $\int \frac{3x^2-2}{x^3-2x-8}\,dx = \int \frac{1}{t}\,dt$

$t = x^3-2x-8$

$\frac{dt}{dx} = 3x^2-2$

$dt = (3x^2-2)dx$

$= \ln|t| + C$

$= \boxed{\ln|x^3-2x-8| + C}$ ✓

This one is tricky since the first impulse maybe to try to factor x^3-2x-8.

27) $\int \frac{dx}{1+e^x}$

let $u = 1+e^x \Rightarrow e^x = u-1$

$du = e^x dx$

$du = (u-1)dx$

$\frac{1}{u-1} du = dx$

$$\int \frac{1}{1+e^x} dx = \int \frac{1}{u} \cdot \frac{1}{u-1} du$$

$$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1} \Rightarrow 1 = A(u-1) + B(u)$$

$u=1$ $1=B$

$u=0$ $1=-A$

$A=-1$

$$\int \frac{1}{1+e^x} dx = \int \frac{1}{u(u-1)} du = \int \left(-1/u + \frac{1}{u-1}\right) du = -\ln|u| + \ln|u-1| + C$$

$$= -\ln|1+e^x| + \ln|e^x| + C$$

$$= -\ln(1+e^x) + \ln e^x + C$$

$$= \boxed{x - \ln(1+e^x) + C}$$

✓ fine

KRG

28) $\int \sin \sqrt{at}\, dt$

$u = \sqrt{at}$

$\frac{du}{dt} = \frac{1}{2}(at)^{-1/2} = \frac{1}{2\sqrt{at}}$?

$2\,du = \frac{1}{\sqrt{at}} dt$ Lost!

OK ~~Since~~ Since $u = \sqrt{at}$

$u^2 = at$

$2u\,du = a\,dt$

$dt = \frac{1}{a} 2u\,du$

parts $w = u$ $dv = \sin u\,du$

$dw = du$ $v = -\cos u$

$$\int \sin\sqrt{at}\, dt = \int \sin u \ \frac{1}{a} 2u\,du = \frac{2}{a}\int u \sin u\, du = \frac{2}{a}\left(-u\cos u + \int \cos u\,du\right)$$

$$= \frac{2}{a}(-u\cos u + \sin u) + C = \boxed{\frac{2}{a}\left(\sqrt{at}\cos\sqrt{at} + \sin\sqrt{at}\right) + C}$$

answer

Group 7 8:00 AM

53. $\int x^2 \sinh(mx)\,dx = \frac{1}{m}x^2\cosh(mx) - \frac{2}{m}\int x\cosh(mx)\,dx$

$u = x^2$ $\quad dv = \sinh(mx)\,dx$

$du = 2x\,dx$ $\quad v = \frac{1}{m}\cosh(mx)$

$= \frac{1}{m}x^2\cosh(mx) - \frac{2}{m}\left(\frac{1}{m}x\sinh(mx) - \frac{1}{m}\int \sinh(mx)\,dx\right)$

$u = x$ $\quad dv = \cosh(mx)\,dx$

$du = dx$ $\quad v = \frac{1}{m}\sinh(mx)$

$$\boxed{= \frac{1}{m}x^2\cosh(mx) - \frac{2}{m^2}x\sinh(mx) + \frac{2}{m^3}\cosh(mx) + C}$$

✓ fine KRG

Group 7

54 $\int (x+\sin x)^2 dx$

multiply out

error

$= \int x^2 + 2x\sin x + \sin^2 x \, dx$

$= \underbrace{\int x^2 dx}_{\boxed{1}} + 2\underbrace{\int \sin x \, dx}_{\boxed{2}} + \underbrace{\int \sin^2 x \, dx}_{\boxed{3}}$

$\boxed{1} = \int x^2 dx = \frac{1}{3}x^3$ ✓

$\boxed{2} = 2\int x\sin x \, dx = 2(-\cos x) = -2\cos x$

$\boxed{3} = \int \sin^2 x \, dx = \int (\sin x)^2 dx = \frac{1}{3}\sin^3 x$ X

$\boxed{1} + \boxed{2} + \boxed{3} = \frac{1}{3}x^2 - 2\cos x + \frac{1}{3}\sin^3 x + C$

$\boxed{1}$ correct

$\boxed{2}$ wrong also

$\boxed{3}$ wrong $\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C$

$\boxed{2}$ $\int x \sin x \, dx$ is an integration by parts

$u = x$ | $dv = \sin x \, dx$

$du = dx$ | $v = -\cos x$

$\Rightarrow = -x\cos x + \int \cos x \, dx = -x\cos x + \sin x$

$\therefore \boxed{1} + \boxed{2} + \boxed{3} = \boxed{\frac{1}{3}x^3 + 2\left[-x\cos x + \sin x\right] + \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C}$

answer KRG

Group 7

$\boxed{55}$ $\int \frac{1}{x+x\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2}x^{-1/2}dx$

$= \frac{1}{2\sqrt{x}}dx$

$dx = 2\sqrt{x}\,du$

$\frac{1}{2x}\int \frac{1}{u^2+(u^2)(u)}du$

variable x outside integral is TROUBLE!

$= \frac{1}{2x}\int \frac{1}{u^2+u^3}du$

$= \frac{1}{2x}\int \frac{1}{u^2(1+u)}du$

$v = 1+u \qquad u = v-1$

$dv = du$

$I = \frac{1}{2x}\int \frac{1}{v(v-1)^2}dv$

$\frac{A}{v} + \frac{B}{v-1} + \frac{C}{(v-1)^2}$

$1 = Av - A + Bv(v-1) + Cv(v-1)$

$1 = Av - A + Bv^2 - Bv + Cv^2 - Cv$

$1 = v^2(B+C) + v(A-B-C) - A$

$B+C=0 \qquad A=1$

$A-B-C=0 \qquad B=-\frac{1}{2}$

$-A=1 \qquad C=\frac{1}{2}$

$= \int \frac{1}{v} - \int \frac{\frac{1}{2}}{v-1} + \int \frac{\frac{1}{2}}{(v-1)^2}$

$= -\ln|v| - \frac{1}{2}\ln|v-1| - \frac{1}{2}\frac{1}{(v-1)} + C$

$= -\ln|1+u| - \frac{1}{2}\ln|1+u-1| - \frac{1}{2(1+u-1)} + C$

$= -\ln|\sqrt{x}| - \frac{1}{2}\ln|\sqrt{x}| - \frac{1}{2\sqrt{x}} + C$

$q = \int \frac{1}{u^2+u^2\cdot u}\,2u\,du$

$= \int \frac{2}{u(1+u)} = I$

after partial fractions

$= \int\left(\frac{2}{u} - \frac{2}{1+u}\right)du$

$= 2u\,du$

$= 2\ln|u| - 2\ln|1+u| + C$

$= \boxed{2\ln\sqrt{x} - 2\ln(1+\sqrt{x}) + C}$

Answer

Note: $\frac{1}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u}$

$1 = A(1+u) + Bu$

$A = 1$

$B = -1$

Group 7

81 $\int \sqrt{1-\sin x}\, dx$

$= \int (1-\sin x)^{1/2} dx$

$= \cancel{\frac{2}{3}(1-\sin x)^{3/2} + C}$

Let $u = \sin x$

$du = \cos x\, dx = \sqrt{1-\sin^2 x}\, dx = \sqrt{1-u^2}\, dx$

so $\frac{1}{\sqrt{1-u^2}}\, du = dx$

$$\int (1-\sin x)^{\frac{1}{2}} dx = \int \sqrt{1-u}\, \frac{1}{\sqrt{1-u^2}}\, du$$

$$= \int \frac{1}{\sqrt{1+u}}\, du$$

$$= 2\sqrt{1+u} + C$$

$$= \boxed{2\sqrt{1+\sin x} + C}$$

answer

Group 7

82 $\displaystyle\int \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\,dx = \int \frac{\sin x\cos x\,dx}{\sin^4 x+(1-\sin^2 x)^2}$

$u = \sin x$

$du = \cos x\,dx$

$= \displaystyle\int \frac{u\,du}{u^4+(1-u^2)^2} = \int \frac{u}{2u^4-2u^2+1}\,du = \int \frac{u}{\cancel{(2u^2-1)^2+1}}\,du$

$= \displaystyle\int \frac{u}{u^4-u^2+1}\,du$

$= \frac{1}{2}\ln\left|u^4-u^2+1\right|+C$

$\times = \frac{1}{2}\ln\left|\sin^4 x-\sin^2 x+1\right|+C$

$= \displaystyle\frac{1}{2}\int \frac{u}{u^4-u^2+\frac{1}{2}}\,du$

$= \displaystyle\frac{1}{2}\int \frac{u}{u^4-u^2+\frac{1}{4}+\frac{1}{4}}$

$= \displaystyle\frac{1}{2}\int \frac{u}{(u^2-\frac{1}{2})^2+\frac{1}{4}}\,du$

$= \displaystyle\frac{1}{4}\int \frac{2u}{(u^2-\frac{1}{2})^2+\frac{1}{4}}\,du$

$w = u^2-\frac{1}{2}$

$dw = 2u\,du$

$= \displaystyle\frac{1}{4}\int \frac{1}{w^2+\frac{1}{4}}\,dw$

$= \frac{1}{4}\cdot\frac{1}{\frac{1}{2}}\arctan\left(\frac{w}{\frac{1}{2}}\right)+C$

$= \frac{1}{2}\arctan(2w)+C = \frac{1}{2}\arctan(2u^2-1)+C$

$= \boxed{\frac{1}{2}\arctan(2\sin^2 x-1)+C}$

Here is the answer