Section 7.5 homework

Clay Sexton
Group 6
6.3.2012

21) $\int \arctan\sqrt{x}\,dx = \int \arctan s\; 2s\,ds$

$s = \sqrt{x}$ $\quad = 2\int s \arctan s\,ds$

$ds = \frac{1}{2\sqrt{x}}dx$ $\quad u = \arctan s \quad dv = s\,ds$

$ds = \frac{1}{2}s\,dx$ $\quad du = \frac{1}{1+s^2}ds \quad v = \frac{s^2}{2}$

$2ds = s\,dx$] no

$ds = \frac{1}{2s}dx$

$2s\,ds = dx$

$= uv - \int v\,du$ etc.

$= 2\left[\frac{s^2}{2}\arctan s - \int \frac{s^2}{2}\frac{1}{1+s^2}ds\right]$

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}\int s^2 \frac{1}{1+s^2}ds\right]$

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}\int (s^2+1-1)\frac{1}{1+s^2}ds\right]$ use parentheses!

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}\int \frac{s^2+1}{1+s^2} - \frac{1}{1+s^2}ds\right]$

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}\int 1 - \frac{1}{1+s^2}ds\right]$

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}s + \frac{1}{2}\arctan s\right]$

$= s^2\arctan s - s + \arctan s + c$

$= (\sqrt{x})^2\arctan\sqrt{x} - \sqrt{x} + \arctan\sqrt{x} + c$

$= x\arctan\sqrt{x} - \sqrt{x} + \arctan\sqrt{x} + c$

$\boxed{(x+1)\arctan\sqrt{x} - \sqrt{x} + c}$

Please write larger!

LSG

(out of order)

49) $\int \frac{1}{x\sqrt{4x+1}}dx = \int \frac{1}{\frac{u^2-1}{4}\cancel{u}}\cancel{u}\,4\,du$

$u = \sqrt{4x+1}$

$x = \frac{u^2-1}{4}$ $\quad = \int \frac{1}{u^2-1}du = \int \frac{1}{(u+1)(u-1)} = \frac{A}{u+1} + \frac{B}{u-1}$

$4u\,du = \cancel{4}dx$

$1 = Au + Bu - A + B$

$A = -B \quad A = (1+A) \quad 2A = -1 \quad A = -\frac{1}{2}$

$1 = -A + B \quad B = 1 + A \quad B = \frac{1}{2}$

$\int \frac{-\frac{1}{2}}{(u+1)} + \frac{\frac{1}{2}}{(u-1)}du$

$= \ln\left|\frac{(u-1)}{(u+1)}\right| + c = \boxed{\ln\left|\frac{(\sqrt{4x+1})-1}{(\sqrt{4x+1})+1}\right| + c}$

graded on other sheet

Mackenzie Skrba

Group 6

02 JUL 12

7.5 → 22, 50, 78

22.) $\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}\,dx$

$\int \ln^2 x\,dx + \int \frac{\ln x}{x}\,dx$, no

$u = \ln x \quad du = \frac{1}{x}\,dx$

$= \int u\,du + \int \ln^2 x\,dx$

$f = \ln^2 x \quad dg = dx$

$df = \frac{2\ln x}{x}\,dx \quad g = x$

$= \int u\,du + x\ln^2 x - 2\int \ln x\,dx$

$f = \ln x \quad dg = dx$

$df = \frac{1}{x}\,dx \quad g = x$

$= \int u\,du + x\ln^2 x - 2x\ln x + 2\int 1\,dx$

$= \int u\,du + 2x + x\ln^2 x - 2x\ln x$

$= \frac{u^2}{u} + 2x + x\ln^2 x - 2x\ln x + C$

$= 2x + x\ln^2 x + \frac{\ln^2 x}{2} - 2x\ln x + C$

$\boxed{= 2x + \left(x+\frac{1}{2}\right)\ln^2 x - 2x\ln x + C}$

Let $u = 1+(\ln x)^2$.

Then

$du = \frac{2\ln x}{x}\,dx$

So $\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}\,dx$

$= \frac{1}{2}\int u^{-1/2}\,du$

$= u^{1/2} + C$

$= \sqrt{1+(\ln x)^2} + C$

LSG

Section 7.5

William Taylor
Group 6

23. $\int_0^1 (1+\sqrt{x})^8\,dx$

$$\left[\begin{array}{l} u=\sqrt{x} \\ du=\frac{1}{2}x^{-\frac{1}{2}}dx \\ dx=2\sqrt{x}\,du \end{array}\right.$$

$$\int (1+u)^8\,2u\cdot du = 2\int (1+u)^8 u\,du$$

$$\left[\begin{array}{l} w=1+u \rightarrow u=w-1 \\ dw=du \end{array}\right.$$

$$\Rightarrow 2\int w^8(w-1)\,dw = 2\int w^9-w^8\,dw = 2\int w^9\,dw - 2\int w^8\,dw$$

$$= \frac{w^{10}}{5} - \frac{2w^9}{9} = \frac{(1+\sqrt{x})^{10}}{5} - \frac{2(1+\sqrt{x})^9}{9}$$

$$\Rightarrow \left.\frac{(1+\sqrt{x})^{10}}{5}\right|_0^1 - \left.\frac{2(1+\sqrt{x})^9}{9}\right|_0^1 = \left(\frac{1024}{5}-\frac{1}{5}\right)-\left(\frac{1024}{9}-\frac{2}{9}\right) \checkmark$$

$$= \underbrace{\frac{1023}{5} - \frac{1022}{9}}_{\text{better as one fraction}} \approx \boxed{91.0444}$$

Don't use decimal approximations

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Group 6
Alarin Taylor

(24) $\int_0^4 \frac{6z+5}{2z+1}\,dz$

$$2z+1\,\overline{)\,6z+5}\quad 3$$
$$-(6z+3)$$
$$2$$

$\int_0^4 \frac{2}{2z+1}+3\,dz$ ✓

$\int_0^4 3_{dz} + 2\int_0^4 \frac{1}{2z+1}\,dz$

$u=2z+1$
$du=2dz$

$dz=\frac{du}{2}$

$12+2\int_0^4 \frac{1}{u}\cdot\frac{du}{2}$ ** *

These limits of integration apply to z, not to u, so don't use them for u.

$12+\int_0^4 \frac{1}{u}\,du$

$12+\ln u\Big|_0^4$

$12+\ln(2z+1)\Big|_0^4$ ✓

$12+\ln(9)\approx 14.1972$

keep answer exact: $\underline{12+\ln 9}$

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$ds = \frac{1}{2\sqrt{x}}dx$ $\quad u = \arctan s \quad dv = s\,ds$

$ds = \frac{1}{2}s\,dx$ $\quad du = \frac{1}{1+s^2}ds \quad v = \frac{s^2}{2}$

$2ds = s\,dx$

$= uv - \int v\,du$

$= 2\left[\frac{s^2}{2}\arctan s - \int \frac{s^2}{2}\frac{1}{1+s^2}ds\right]$

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}\int s^2 \frac{1}{1+s^2}ds\right]$

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}\int s^2+1-1 \frac{1}{1+s^2}ds\right]$

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}\int \frac{s^2+1}{1+s^2} - \frac{1}{1+s^2}ds\right]$

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}\int 1 - \frac{1}{1+s^2}ds\right]$

$= 2\left[\frac{s^2}{2}\arctan s - \frac{1}{2}s + \frac{1}{2}\arctan s\right]$

$= s^2\arctan s - s + \arctan s + C$

$= (\sqrt{x})^2\arctan\sqrt{x} - \sqrt{x} + \arctan\sqrt{x} + C$

$= x\arctan\sqrt{x} - \sqrt{x} + \arctan\sqrt{x} + C$

$\boxed{(x+1)\arctan\sqrt{x} - \sqrt{x} + C}$

graded on earlier page

49) $\int \frac{1}{x\sqrt{4x+1}}dx = \int \frac{1}{\frac{u^2-1}{4}u}\,4du$

$u = \sqrt{4x+1}$

$x = \frac{u^2-1}{4}$

$4du = dx$

very hard to read!

no

$u^2 = 4x+1$

$2u\,du = 4dx$

$\frac{1}{2}u\,du = dx$

$= 2\int \frac{1}{u^2-1}du = 2\int \frac{1}{(u+1)(u-1)} \overset{no}{=} \frac{A}{u+1} + \frac{B}{u-1}$

$1 = Au + Bu - A + B$

$A = -B \quad A = (1+A) \quad 2A = -1 \quad A = -\frac{1}{2}$ ✓

$1 = -A + B \quad B = 1 + A \quad B = \frac{1}{2}$ ✓

$\int \frac{-\frac{1}{2}}{(u+1)} + \frac{\frac{1}{2}}{(u-1)}du$

$\frac{2}{2}\ln\left|\frac{(u-1)}{(u+1)}\right| + C = \boxed{\ln\left|\frac{(\sqrt{4x+1})-1}{(\sqrt{4x+1})+1}\right| + C}$

$\int \frac{1}{x\sqrt{4x+1}}dx = \int \frac{4}{u^2-1}\cdot\frac{1}{u}\cdot\frac{1}{2}u\,du = 2\int\frac{1}{u^2-1}du = 2\int\frac{1}{(u+1)(u-1)}du$

$= 2\int\left[-\frac{1}{2}\left(\frac{1}{u+1}\right) + \frac{1}{2}\left(\frac{1}{u-1}\right)\right]dx =$

$-\ln|u+1| + \ln|u-1| + C$

$= \ln\left|\frac{u+1}{u-1}\right| + C =$

LSG

50.) $\int \frac{1}{x^2\sqrt{4x+1}}\,dx = \frac{1}{10 \times 2^{2/5}}$

$-4 \ln\left(\sqrt[5]{2}\,\sqrt{x}+1\right) + (1+\sqrt{5}) \log\left(2^{2/5} x + \frac{(\sqrt{5}-1)\sqrt{x}}{2^{4/5}} + 1\right) - (\sqrt{5}-1) \ln\left(2^{2/5} x - \frac{(1+\sqrt{5})\sqrt{x}}{2^{4/5}} + 1\right) + 2\sqrt{2(5+\sqrt{5})}\tan^{-1}\left(\frac{4\sqrt[5]{2}\sqrt{x}-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right) - 2\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{4\sqrt[5]{2}\sqrt{x}+\sqrt{5}-1}{\sqrt{2(5+\sqrt{5})}}\right)\Bigg) + C$

What CAS system did you use?
You need to be able to justify your answer.
Compare this to exercise 49.

50 $\int \frac{1}{x^2\sqrt{4x+1}}\,dx = \int \frac{1}{\frac{(u^2-1)^2}{16}\,u}\,\frac{1}{2}u\,du$

$u = \sqrt{4x+1}$

$u^2 = 4x+1$

$\frac{u^2-1}{4} = x$

$\frac{(u^2-1)^2}{16} = x^2$

$dx = \frac{1}{2}u\,du$

$= 8\int \frac{1}{(u^2-1)^2}\,du = 8\int \frac{1}{(u+1)^2(u-1)^2}\,du$

$= 8\int \frac{A}{u+1} + \frac{B}{(u+1)^2} + \frac{C}{u-1} + \frac{D}{(u-1)^2}\,du = I$

$\therefore 1 = A(u+1)(u-1)^2 + B(u-1)^2 + C(u+1)^2(u-1) + D(u+1)^2$

$u=1 \quad 1 = 4D \Rightarrow D = \frac{1}{4}$

$u=-1 \quad 1 = 4B \Rightarrow B = \frac{1}{4}$

u^3 coef $\quad 0 = A+C$

constants $\quad 1 = A + B - C + D \Rightarrow \frac{1}{2} = A - C$

$2A = \frac{1}{2}$

$A = \frac{1}{4}$

$C = -\frac{1}{4}$

$\therefore I = 8\int \frac{1}{4}\frac{1}{u+1} + \frac{1}{4}\frac{1}{(u+1)^2} - \frac{\frac{1}{4}}{u-1} + \frac{\frac{1}{4}}{(u-1)^2}\,du$

$= 2\ln|u+1| - \frac{2}{u+1} - 2\ln|u-1| - \frac{2}{u-1} + C$

$= \boxed{2\ln(\sqrt{4x+1}+1) - \frac{2}{\sqrt{4x+1}+1} - 2\ln|\sqrt{4x+1}-1| - \frac{2}{\sqrt{4x+1}-1} + C}$ answer

GROUP 6

(51) $\int \frac{1}{x\sqrt{4x^2+1}}\,dx$

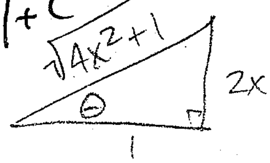
$$\left[\begin{array}{l} 2x = \tan\theta, \quad -\frac{\pi}{2} < \theta \frac{\pi}{2} \\ x = \frac{1}{2}\tan\theta \\ dx = \frac{1}{2}\sec^2\theta\, d\theta \end{array}\right.$$

$$\Rightarrow \int \frac{1}{\cancel{\tfrac{1}{2}}\tan\theta\sqrt{4(\frac{1}{2}\tan\theta)^2+1}} \cdot \cancel{\tfrac{1}{2}}\sec^2\theta\, d\theta$$

$$= \int \frac{\sec^2\theta}{\tan\theta\sqrt{\tan^2\theta+1}}\,d\theta = \int \frac{\sec^{\cancel{2}}\theta}{\tan\theta\,\cancel{\sec\theta}}\,d\theta = \int \frac{\sec\theta}{\tan\theta}\,d\theta$$

$$= \int \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\sin\theta}\,d\theta = \int \csc\theta\, d\theta$$

$$* \quad = \underbrace{-\ln\left|\csc\theta + \cot\theta\right|}_{\ln\left|\csc\theta - \cot\theta\right| + C} + C = -\ln\left|\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right| + C$$



$$\Rightarrow -\ln\left|\frac{\sqrt{4x^2+1}}{2x} + \frac{1}{2x}\right| + C = \boxed{-\ln\left|\frac{1+\sqrt{4x^2+1}}{2x}\right| + C}$$

$$\ln\left|\frac{\sqrt{4x^2+1}}{2x} - \frac{1}{2x}\right| + c$$

$$= \ln\left|\frac{\sqrt{4x^2+1}-1}{2x}\right| + c$$

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Group 6
Alain Taylor

(52) $\int \frac{dx}{x(x^4+1)}$

$u = x^4$
$du = 4x^3 dx$

$dx = \frac{du}{4x^3}$ or let $u = x^2 \ldots$

$\int \frac{du}{4x^3 \cdot x(u+1}$

$\frac{1}{4}\int \frac{du}{u(u+1)}$

$1 = \frac{A}{u} + \frac{B}{u+1}$

$A(u+1) + Bu$

$A = 1$
$B = -1$

$\frac{1}{4}\int \frac{1}{u} du - \frac{1}{4}\int \frac{1}{u+1} du$

$\frac{1}{4}\ln u - \frac{1}{4}\int \frac{1}{u+1} du$

$\frac{\ln x^4}{4} - \frac{1}{4}\int \frac{1}{u+1} du$

$w = u+1$
$dw = du$

$\int \frac{1}{w} dw$

$\frac{\ln x^4}{4} - \frac{\ln w}{4}$

$\frac{1}{4}\left(\ln x^4 - \ln(x^4+1)\right) + C$

LSG

I =

77) $\int \frac{xe^x}{\sqrt{1+e^x}}dx$

$u = 1+e^x$

$du = e^x dx$

$x = \ln(u-1)$

$\int \frac{\ln(u-1)}{\sqrt{u}}du = \int \frac{1}{\sqrt{u}}\ln(u-1)du$

$= 2\int \ln(u-1)\, d(\sqrt{u})$

$= 2\sqrt{u}\ln(u-1) - \boxed{2\int \frac{\sqrt{u}}{u-1}du}$

$= 2\sqrt{u}\ln(u-1) - \boxed{4\int \frac{(\sqrt{u})^2}{[(\sqrt{u})^2-1]}d(\sqrt{u})}$

how is an integral of a function suddenly equal to the integral of the square of the function

$= 2\sqrt{u}\ln(u-1) - 4\int \frac{[\sqrt{u}^2-1]}{[(\sqrt{u})^2-1]}d\sqrt{u} - 4\int \frac{1}{[(\sqrt{u})^2-1]}\frac{du}{d\sqrt{u}}$

$= 2\sqrt{u}\ln(u-1) - 4\sqrt{u} - 4\int \frac{1}{\sqrt{u}^2-1}d(\sqrt{u})$

$= 2\sqrt{u}\ln(u-1) - 4\sqrt{u} - 2\underbrace{\left[\int \frac{1}{\sqrt{u}-1}d(\sqrt{u}) - \int \frac{1}{\sqrt{u}+1}d(\sqrt{u})\right]}_{\text{show work}}$

Also, $(u-1)^2 \neq u^2-1$

show work for getting A & B (ok - done on prev. pb.)

$= 2\sqrt{u}\ln(u-1) - 4\sqrt{u} - 2\left(\frac{\ln(\sqrt{u}-1)}{\ln(\sqrt{u}+1)}\right) + C$

careful! →

some errors here

$= 2\sqrt{(1+e^x)}\ln(\cancel{(1+e^x)-1}) - 4(\sqrt{1+e^x}) - 2\ln\left|\frac{(\sqrt{1+e^x}-1)}{(\sqrt{1+e^x}+1)}\right| + C$

$2x\sqrt{1+e^x} \quad - 4\sqrt{1+e^x}$

$= \boxed{2(x-2)\sqrt{1+e^x} + 2\ln\left|\frac{(\sqrt{1+e^x}-1)}{(\sqrt{1+e^x}+1)}\right| + C}$ right answer but

So little work is shown that it is hard to follow your work!

I would let $w = \sqrt{1+e^x}$

$w^2 = 1+e^x$

$w^2-1 = e^x \Rightarrow x = \ln(w^2-1)$

$2w\,dw = e^x dx = (w^2-1)dx$

$dx = \frac{2w}{w^2-1}dw$

$\therefore I = \int \frac{\ln(w^2-1)}{w}\frac{2w}{w^2-1}dw = 2\int \ln(w+1)dw + 2\int \ln(w-1)dw$

$= 2\left[(w+1)\ln(w+1) - (w+1) + (w-1)\ln(w-1) - (w-1)\right] + C$

$= 2\left[w\ln(w^2-1) + \ln\frac{w+1}{w-1} - 2w\right] + C$

$= 2\left[x\sqrt{1+e^x} + \ln\boxed{\frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}}\right] - 2\sqrt{1+e^x} + C$

78) $\int \frac{1+\sin x}{1-\sin x}\,dx$

should this be 1? I can't get a two here.

$u = \tan\left(\frac{x}{2}\right) \quad du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$

$= \int \frac{2\left(\frac{2u}{u^2+1}+1\right)}{(u^2+1)\left(1-\frac{2u}{u^2+1}\right)}\,du$

So you are using Weierstrass substitutions mentioned in #59 page 493 which will work.

$= \int \frac{2(u+1)^2}{u^4-2u^3+2u^2-2u+1}\,du$

$= 2\int \frac{(u+1)^2}{u^4-2u^3+2u^2-2u+1}\,du$

Here is another approach that avoids Weierstrass.

$= 2\int \frac{(u+1)^2}{(u-1)^2(u^2+1)}\,du$

$\frac{1+\sin x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}$

$= 2\int \left(\frac{2}{(u-1)^2} - \frac{1}{u^2+1}\right)du$

$= \frac{1+2\sin x+\sin^2 x}{1-\sin^2 x}$

$= 4\int \frac{1}{(u-1)^2}\,du - 2\int \frac{1}{u^2+1}\,du$

$= \frac{1+2\sin x+\sin^2 x}{\cos^2 x}$

$= 4\int \frac{1}{(u-1)^2}\,du - 2\tan^{-1}(u)$

$= \sec^2 x + 2\sec x\tan x + \sec^2 x - 1$

$= 2\sec^2 x + 2\sec x\tan x - 1$

$= 4\int \frac{1}{s^2}\,ds - 2\tan^{-1}(u)$

the integral of the above function is $\boxed{2\tan x + 2\sec x - x + C}$

$= -\frac{4}{s} - 2\tan^{-1}(u) + C$

$= \frac{-2(u-1)\tan^{-1}(u) - 4}{u-1} + C$

This is easier to work with than

$= \frac{-2\tan^{-1}\left(\tan\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)-1\right)-4}{\tan\left(\frac{x}{2}\right)-1} + C$

$\boxed{= \frac{(x+4)\sin\left(\frac{x}{2}\right) - x\cos\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)} + C}$

GROUP 6

(79) $\int x \sin^2 x \cos x \, dx$

$$\left[\begin{array}{ll} u = x & dv = \sin^2 x \cos x \, dx \\ du = dx & v = \int \sin^2 x \cos x \, dx \end{array}\right.$$

$$\left[\begin{array}{l} w = \sin x \\ dw = \cos x \, dx \end{array}\right.$$

$$\Rightarrow v = \int w^2 \, dw = \frac{w^3}{3} = \frac{\sin^3 x}{3}$$

$$\Rightarrow uv - \int v \, du = \frac{x \sin^3 x}{3} - \int \frac{\sin^3 x}{3} \, dx$$

$$= \frac{1}{3} x \sin^3 x - \frac{1}{3} \int \sin x (1 - \cos^2 x) \, dx$$

$$\left[\begin{array}{l} z = \cos x \\ dz = -\sin x \, dx \end{array}\right.$$

$$\Rightarrow \frac{1}{3} x \sin^3 x + \frac{1}{3} \int 1 - z^2 \, dz = \frac{1}{3} x \sin^3 x + \frac{1}{3}\left(z - \frac{z^3}{3}\right) + C$$

$$= \boxed{\frac{1}{3} x \sin^3 x + \frac{1}{3} \cos x - \frac{1}{9} \cos^3 x + C}$$

very nicely done

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Group 6

80 $\displaystyle\int \frac{\sec x \cos 2x}{\sin x + \sec x}\,dx$

Show work : mult thru by $2\cos x$:

$$\int \left(\frac{\sec x \cos(2x)}{\sin x + \sec x}\right)\frac{2\cos x}{2\cos x}\,dx =$$

$$\int \frac{2\cos(2x)}{\underbrace{2\sin x \cos x}_{2\sin(2x)} + 2}\,dx$$

$$\int \frac{2\cos(2x)}{\sin(2x)+2}\,dx$$

$$2\int \frac{\cos 2x}{\sin 2x+2}\,dx$$

$u = 2x$

$du = 2dx$

alt : let $u = \sin(2x) + 2$

then $du = 2\cos 2x\,dx$

$$\int \frac{\cos u}{\sin u+2}\,du$$

$w = \sin u + 2$

$dw = \cos u\,du$

$$\int \frac{1}{w}\,dw$$

$\ln(w) + C$

$\ln(\sin u + 2) + C$

$\ln|\sin 2x + 2| + C$

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