Alexandre Pellegrini
7.5 Problems
Group 5

17. $\int_0^{\pi} t\cos^2 t \cdot dt$ $\qquad$ error $dv = \left(\frac{1+\cos(2t)}{2}\right)dt$

$\int_0^{\pi} t\,\frac{(1+\cos(2t))}{2} \cdot dt$

$u = t$ $\quad dv = \frac{1}{2}\cos(2t)$ $\qquad v = \int \cos(2t)\cdot dt$ $\qquad u = 2t$

$du = dt$ $\quad v = \frac{1}{4}\sin(2t) + \frac{1}{2}t$ $\qquad v = \int \frac{1}{2}\cos(u)\cdot du$ $\qquad du = 2\cdot dt$

$uv - \int v \cdot du$ $\qquad v = \cancel{\frac{1}{2}\sin(2t)}$ $\rightarrow \frac{1}{2}t + \frac{1}{4}\sin(2t)$

$t \cdot \frac{1}{2}\sin(2t) - \int \frac{1}{2}\sin(2t)\cdot dt$

$\frac{t}{2}\sin(2t) + \frac{1}{4}\cos(2t) \Big]_0^{\pi}$

$\frac{\pi}{2}\sin(2\pi) + \frac{1}{4}\cos(2\pi) - \frac{1}{4}$

0.1704

$$uv - \int v\,du = t\left(\frac{1}{2}t + \frac{1}{4}\sin(2t)\right) - \int\left(\frac{1}{2}t + \frac{1}{4}\sin(2t)\right)dt$$

$$= t\left(\frac{1}{2}t + \frac{1}{4}\sin(2t)\right) - \frac{1}{4}t^2 - \frac{1}{8}\cos(2t)$$

$$= \frac{1}{4}t^2 + \frac{1}{4}t\sin(2t) - \frac{1}{8}\cos(2t)$$

$$\left[\frac{1}{4}t^2 + \frac{1}{4}t\sin(2t) - \frac{1}{8}\cos(2t)\right]\Bigg|_0^{\pi}$$

$$= \frac{\pi^2}{4} + \frac{1}{4}\pi\sin(2\pi) - \frac{1}{8}\cos(2\pi)$$
$$- \left[\frac{1}{4}0^2 + \frac{1}{4}0^2\sin(0) - \frac{1}{8}\cos(0)\right]$$

$$= \boxed{\frac{\pi^2}{4}}$$ correct answer

KRG

Christopher Poteat

Group 5

(19) $\int e^{x+e^x}\,dx$

$u = e^x$
$du = e^x$

$= \int e^x \cdot e^{e^x}\,dx$

$= \int du\, e^u$

$= e^u + C$

$= \boxed{e^{e^x}} + C$

Don't forget "dx" + "+ C"

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Group 5

7.5 #20

$\int e^2 dx = \cancel{\int} e^2 x + C$

$du = e^2$

$u = e^2 x + C$

"$\int$" is incorrect since the integration has already ~~been~~ been done

$\therefore$ the answer is $\boxed{e^2 x + c}$

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Alexandre Penegra
7.5 Problems
Group 5

45. $\int x^5 e^{-x^3} \cdot dx$

$\int x^3 \cdot x^2 \cdot e^{-x^3} \cdot dx$

$u = x^3 \quad du = 3x^2 \cdot dx$

$\int \frac{1}{3} u \cdot e^{-u} \cdot du$

$z = u \quad dv = e^{-u}$

$dz = du \quad v = e^{-u}$ X $\boxed{v = -e^{-u}}$

$zv - \int v\,dz$

$\frac{1}{3}\left[-(u)(e^{-u}) + \int e^{-u} \cdot du\right]$

$\cancel{e^{-u}(u-1)+c}$ $\frac{1}{3}\left[u e^{-u} - e^{-u}\right] + c$

$e^{-x^3}(x^3-1) + C$ X Answer is $-\frac{1}{3}\left[x^3 e^{-x^3} + e^{-x^3}\right] + C$

$= \boxed{-\frac{1}{3} e^{-x^3}\left[x^3 + 1\right] + C}$

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Group 5

(47) $\int x^3 (x-1)^{-4}\,dx = \int \frac{x^3}{(x-1)^4} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{(x-1)^4}$

$A=1 \quad C=3$

$B=3 \quad D=1$

Correct but How?

$= \int \frac{1}{x-1} + 3\int \frac{1}{(x-1)^2} + 3\int \frac{1}{(x-1)^3} + \int \frac{1}{(x-1)^4}$

$u = x-1$

$du = dx$

$= \int \frac{du}{u} + \frac{3}{2}\int \frac{du}{u^2} + 3\int \frac{du}{u^3} + \int \frac{du}{u^4}$

$= \ln|u| - 3u^{-1} - \frac{3}{2}u^{-2} - \frac{1}{3}u^{-3} + C$

$= \boxed{\ln|x-1| - 3(x-1)^{-1} - \frac{3}{2}(x-1)^{-2} - \frac{1}{3}(x-1)^{-3} + C}$ ✓

this answer above is correct —

Another approach is to use $u = x-1$; $du = dx$

then $\int x^3(x-1)^{-4}\,dx = \int (u+1)^3 u^{-4}\,du = \int (u^{-1} + 3u^{-2} + 3u^{-3} + u^{-4})\,du$

$= \ln|u| - 3u^{-1} - \frac{3}{2}u^{-2} - \frac{1}{3}u^{-3} + C$

$= \ln|x-1| - 3(x-1)^{-1} - \frac{3}{2}(x-1)^{-2} - \frac{1}{3}(x-1)^{-3} + C$

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7.5 # 48

$$\int_0^1 x\sqrt{2-\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int_1^0 \sqrt{2-\sqrt{u}}\,du = -\int_1^0 \sqrt{2-v}\,v\,dv = -\int_1^2 (q-2)\sqrt{q}\,dq$$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$v = \sqrt{u}$

$dv = \frac{1}{2\sqrt{u}}du$

$2v\,dv = du$

$q = 2-v$

$dq = -dv$

$v = 2-q$

$$= -\int_1^2 (q^{3/2} - 2\sqrt{q})\,dq = 2\int_1^2 \sqrt{q}\,dq - \int_1^2 q^{3/2}\,dq$$

this process is correct; just need to finish as is done here. KRG

$$= 2\frac{q^{3/2}}{3/2} - \frac{q^{5/2}}{5/2}\Big|_1^2$$

$$= \left(\frac{4}{3}q^{3/2} - \frac{2}{5}q^{5/2}\right)\Big|_1^2$$

$$= \frac{4}{3}2^{3/2} - \frac{2}{5}2^{5/2} - \left(\frac{4}{3} - \frac{2}{5}\right)$$

$$= \frac{4}{3}2\sqrt{2} - \frac{2}{5}2^2\sqrt{2} - \frac{14}{15}$$

$$= \frac{8}{3}\sqrt{2} - \frac{8}{5}\sqrt{2} - \frac{14}{15}$$

$$= \boxed{\frac{16}{15}\sqrt{2} - \frac{14}{15}} \leftarrow \text{Answer}$$

Alexandre Peregrino
7.5 Problems

73. $\int \frac{x + \arcsin x}{\sqrt{1-x^2}} \, dx$

$u = \arcsin x$

$du = \frac{1}{\sqrt{1-x^2}} \, dx$

$\int \sin(u) + u$

$-\cos(u) + \frac{1}{2}u^2 + C$

$-\cos(\sin^{-1}x) + \frac{1}{2}(\sin^{-1}x)^2 + C$

$-\sqrt{1-x^2} + \frac{1}{2}(\sin^{-1}x)^2 + C$

Answer correct with "+ C"

Christopher Poteat

Group 5

75) $\int \frac{1}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4}$

$1 = A(x^2+4) + (Bx+C)(x-2)$

Show how A, B, C are derived

$A = \frac{1}{8}$ $B = -\frac{1}{8}$ ~~C = 2~~ $C = -\frac{1}{4}$

$1 = (A+B)x^2 + (-2B+C)x + \boxed{4A - 2C}$

$= \int \frac{1}{8(x-2)} + \frac{-x-2}{8(x^2+4)}$

? Here you are using $C = -\frac{1}{4}$

$= \frac{1}{8}\int \frac{1}{x-2} \cancel{+} - \frac{1}{16}\int \frac{2x}{x^2+4} - \frac{1}{8}\int \frac{2}{x^2+4}$

$= \boxed{\frac{1}{8}\ln|x-2| - \frac{1}{16}\ln|x^2+4| - \frac{1}{8}\tan^{-1}\left(\frac{x}{2}\right) + C}$

Your answer is correct but the preceding line has errors

KRG

7.5 #76

Group 5

$$\int \frac{dx}{\sqrt{x}(2+\sqrt{x})^4} = 2\int \frac{1}{u^4}\,du = -\frac{2}{3u^3} = -\frac{2}{3(\sqrt{x}+2)^3} + C$$

$u = \sqrt{x}+2$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

Answer and procedure are fine — KRG