Group 4

$\boxed{13.}$ $\int \sin^5 t \cos^4 t \, dt$

$\int [\sin^4 t \cos^4 t \sin t] \, dt$

$\int (1-\cos^2(t))^2 \cos^4(t) \sin(t) \, dt$

$u = \cos(t)$

$du = -\sin(t) \, dt$

$-\int (1-u^2)^2 u^4 \, du$

$-\int (1-2u^2+u^4) u^4 \, du$

$-\int u^4 - 2u^6 + u^8 \, du$

$-\left[\frac{u^5}{5} - \frac{2u^7}{7} + \frac{u^4}{9}\right] + C = \boxed{-\frac{\cos^5(t)}{5} + \frac{2\cos^7(t)}{7} + \frac{\cos^9(t)}{9} + C}$

9

9

Hard to tell it's a 9.

fine ✓

KRB

Group 4

$\boxed{\#15}$ $\displaystyle\int \frac{dx}{(1-x^2)^{3/2}}$ $\quad a=1$, $x=x$ $\quad \sqrt{a^2-x^2} \Rightarrow x = a\sin\theta$

$\downarrow$

$\displaystyle\int \frac{1}{(\sqrt{1-x^2})^3}\,dx$

$x=\sin\theta$

$\frac{dx}{d\theta}=\cos\theta$

$dx=\cos\theta\, d\theta$

$\sqrt{1-x^2} \rightarrow \sqrt{1-\sin^2\theta}$

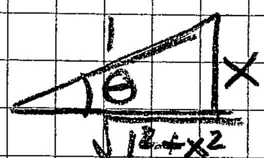
$=\sqrt{\cos^2\theta}$

$=\cos\theta$

$\downarrow$

$\displaystyle\int \frac{1}{(\cos\theta)^3}\,dx \rightarrow \int \frac{\cancel{\cos\theta}\, d\theta}{\cos^{\cancel{3}2}\theta} \rightarrow \int \frac{1}{\cos^2\theta}\,d\theta$

$\displaystyle= \int \sec^2\theta\, d\theta \rightarrow \tan\theta + C$



$x=(1)\sin\theta$

$\frac{x}{1}=\sin\theta$

$\tan\theta = \frac{x}{\sqrt{1-x^2}}$

so answer $= \boxed{\dfrac{x}{\sqrt{1-x^2}} + C}$ ✓

fine

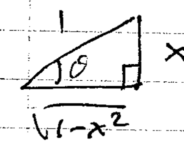
KRG

Group 4

#16)

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}} \quad \text{let } x = \sin\theta \quad dx = \cos\theta\, d\theta \quad = \int_*^{**} \frac{\sin^2\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}\, d\theta$$

illegible "sin"



Limits of integration?

$$= \int_*^{**} \sin^2\theta\, d\theta = \int \frac{1-\cos 2\theta}{2}\, d\theta = \frac{1}{2}\int (1-\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\left[\int d\theta - \int \cos 2\theta\, d\theta\right] = \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right] = \left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_*^{**}$$

$$\frac{1}{4}\sin 2\theta = \frac{1}{4}\, 2\sin\theta\cos\theta = \frac{1}{2} x\sqrt{1-x^2}$$

$$= \boxed{\frac{\sin^{-1}\theta}{2}}^{\text{no}} \quad \frac{1}{2}\left[\frac{1}{2} x\sqrt{1-x^2}\right] \longrightarrow \boxed{\frac{1}{2x} - \frac{1}{4} x\sqrt{1-x^2}}$$

$$\frac{1}{\sin\theta} = \frac{1}{x} =$$

not finished! This was a definite integral.

$$\frac{1}{2}\theta = \frac{1}{2}\sin^{-1}(x)$$

OR Better

If you convert limits of integration in terms of θ,
then if $x = \frac{\sqrt{2}}{2}$, $\theta = \frac{\pi}{4}$ $(x = \sin\theta)$

and if $x = 0$, $\theta = 0$

$$\text{So } \left[\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)\right]_0^{\pi/4} = \left[\frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{4}\sin\left(\frac{\pi}{2}\right)\right] - (0-0)$$

$$= \frac{\pi}{8} - \frac{1}{4}(1) = \frac{\pi}{8} - \frac{1}{4}$$

Group 4

$\boxed{41}$ $\int \theta \tan^2\theta \, d\theta$

$u = \theta$
$du = 1$
$v = \tan\theta - \theta$
$dv = \tan^2\theta$

$= \theta(\tan\theta - \theta) - \int (\tan\theta - \theta)\, d\theta$

$= \theta\tan\theta - \theta^2 - \left(\overset{\text{no}}{\theta} \ln|\sec\theta| - \frac{\theta^2}{2}\right)$

$\theta\tan\theta - \theta^2 \overset{-}{+} \ln|\sec\theta| + \frac{\theta^2}{2} + C$

$$\boxed{\theta\tan\theta - \frac{\theta^2}{2} \overset{-}{+} \ln|\sec\theta| + C}$$

fine
KRV

Group 9

$\boxed{\#43}$ $\displaystyle\int \frac{\sqrt{x}}{1+x^3}\,dx$ $\quad u = \cancel{x}^{3/2}$ $\longrightarrow du = 3/2\, x^{1/2}\,dx$

$u^2 = x^3$ $\quad 2/3\,du = x^{1/2}\,dx$

$$\int \frac{2/3\,du}{1+u^2} \rightarrow \frac{2}{3}\int \frac{du}{1+u^2} \rightarrow \left(\frac{2}{3}\right)\left(\frac{1}{a}\right)\tan^{-1}(u/a)+C$$

$$\frac{2}{3}\left(\frac{1}{1}\right)\tan^{-1}(x^{3/2}/1)+C$$

$$\boxed{= \frac{2}{3}\tan^{-1}(x^{3/2})+C}$$ ✓

fine KRG

Group 4

#44 $\int\sqrt{1+e^x}\,dx$ Let $u=\sqrt{1+e^x}$

$du=\frac{1}{2}(1+e^x)^{-1/2}e^x\,dx$

$2e^{-x}(1+e^x)^{1/2}du = dx$

$u^2 = 1+e^x$ ✓ now differentiate both sides

so $2u\frac{du}{dx} = e^x = u^2-1$

$\therefore \frac{2u}{u^2-1}\,du = dx$

$\int u\,2e^{-x}u\,du = \int\frac{u^2}{2e^x}\,du = \frac{1}{2}\int\frac{u^2}{e^x}\,du$

you can't mix variables like this!

$=\frac{1}{2}\int\frac{1+e^x}{e^x} = \frac{1}{2}\left(\int\frac{1}{e^x}+\int\frac{e^x}{e^x}\right)$

Let $u=\sqrt{1+e^x}$

$\frac{du}{dx}=\frac{e^x}{\sqrt{1+e^x}}$

$du=\frac{e^x\,dx}{\sqrt{1+e^x}}$ $\quad dx=\frac{\sqrt{1+e^x}}{e^x}\,du \Rightarrow dx=\frac{u}{e^x}$ $\quad e^x=u^2-1$

$\int u\left(\frac{u}{e^x}\right)du = \int\frac{u}{u^2-1}\,du = \int\frac{u\,du}{u^2-1}\int\frac{u}{(u+1)(u-1)}\,du =$

$\frac{u}{(u+1)(u-1)} = \frac{A}{(u+1)}+\frac{B}{(u-1)}$ $\quad u = A(u-1)+B(u+1)$

$u=1$ $B=\frac{1}{2}$

$u=-1$ $A=\frac{1}{2}$

$=\int\frac{1}{2(u+1)}+\frac{B}{2(u-1)} = \frac{1}{2}\int\frac{1}{u+1}+\frac{1}{u-1} = \frac{1}{2}(\ln|u+1|+\ln|u-1|)$

$\boxed{=\frac{1}{2}\left(\ln|\sqrt{1+e^x}+1|+\ln|\sqrt{1+e^x}-1|\right)}$

$\int\sqrt{1+e^x}\,dx = \int u\,\frac{2u}{u^2-1}\,du$

$=\int\frac{2u^2}{u^2-1}\,du = \int\left(\frac{2u^2-2}{u^2-1}+\frac{2}{u^2-1}\right)du = \int 2+\frac{2}{(u-1)(u+1)}\,du$

$\int 2\,du + 2\int\left(\frac{A}{u-1}+\frac{B}{u+1}\right)du = \int 2\,du + 2\int\left(\frac{1}{u-1}-\frac{1}{u+1}\right)du$

$\Rightarrow 2u + 2(\ln|u-1| - \ln|u+1|)+C = \boxed{2\sqrt{1+e^x} + 2\ln\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}+C}$

$\tan\theta = 1 \rightarrow \theta = \pi/4$
$\tan\theta = \sqrt{3} \rightarrow \theta = \pi/3$ ✓

Group 4

$\boxed{69}$ $\displaystyle\int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2}$ $x = \tan\theta$, $dx = \sec^2\theta\, d\theta$

$$\int \frac{\sqrt{1+\tan^2\theta}}{\tan^2\theta}\sec^2\theta\, d\theta \rightarrow \int \frac{\sqrt{\sec^2\theta}}{\tan^2\theta}\sec^2\theta\, d\theta$$

$$\int \frac{\sec\theta}{\tan^2\theta}\cdot\sec^2\theta\, d\theta \rightarrow \cancel{\int \frac{\sec^3\theta}{\tan^2\theta}\, d\theta}$$

$$\downarrow$$

$$\int \frac{\sec\theta\cdot(1+\tan^2\theta)}{\tan^2\theta} \rightarrow \int \frac{\sec\theta + \sec\theta\tan^2\theta}{\tan^2\theta}$$

$$\int \frac{\sec\theta}{\tan^2\theta} + \int \cancel{\frac{\sec\tan^2\theta}{\tan^2\theta}} \qquad \int \frac{\sec\theta}{\tan^2\theta} + \int \sec\theta$$

$$\int \cot\theta\csc\theta + \int \sec\theta = -\csc\theta + \ln|\sec\theta+\tan\theta|$$

$$-\csc\theta + \ln|\sec\theta+\tan\theta| \Big]_{\pi/4}^{\pi/3} ✓$$

$$\left[\left(-2/\sqrt{3}\right) + \ln|2+\sqrt{3}|\right] - \left[-\sqrt{2} + \ln|\sqrt{2}+1|\right]$$

$$= \boxed{\sqrt{2} - 2/\sqrt{3} + \ln(2+\sqrt{3}) - \ln(\sqrt{2}-1)}$$

✓

fine
KRG

Group 4

$\boxed{\#71}$ $\int \frac{e^{2x}}{1+e^x}\,dx \rightarrow \int \frac{e^x \cdot e^x}{1+e^x}\,dx$

$u = 1+e^x$

$\frac{du}{dx} = e^x$

$du = e^x\,dx$

$\int \frac{e^x\,du}{u} \longrightarrow \int \frac{u-1}{u}\,du$

$u = 1+e^x$

$u-1 = e^x$

$\int \frac{u-1}{u}\,du = \int 1 - \frac{1}{u}\,du$

$= u - \ln|u| + C$

$\boxed{= (1+e^x) - \ln|1+e^x| + C}$

fine

KRG

Group 4

#72

$$\int \frac{\ln(x+1)}{x^2}\,dx$$

let $u = \ln(x+1)$ $dv = \frac{1}{x^2}$

$= \ln(x+1) \cdot \frac{-1}{x} + \int \frac{1}{x+1} \cdot \frac{1}{x}$

$-\frac{\ln(x+1)}{x} + \int \frac{1}{x(x+1)} \rightarrow \frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{(x+1)} \rightarrow 1 = A(x+1) + Bx$

$x=-1$ $B=-1$

$x=0$ $A=1$

$= \int \frac{1}{x} - \int \frac{1}{(x+1)} = \ln|x| - \ln|x+1|$

$$\boxed{= -\frac{\ln|x+1|}{x} + \ln|x| - \ln|x+1|}$$

fini

KRb