Group 3
Christy Jaggers
Natalie Lynch
Deanna La

9. $\int_1^3 r^4 \ln r \, dr$

$u = \ln r$
$du = \frac{1}{r}\, dr$
$dv = r^4\, dr$
$v = \frac{1}{5} r^5$

$= \frac{1}{5} r^5 \ln r \Big|_1^3 - \int_1^3 \frac{1}{5} r^{\cancel{5}\,4} \cdot \frac{1}{\cancel{r}}\, dr$

$= \frac{1}{5} r^5 \ln r \Big|_1^3 - \frac{1}{5}\int_1^3 r^4 \, dr$

$= \left[\frac{1}{5}(3)^5 \ln 3 - \frac{1}{5}(1)^5 \ln 1\right] - \frac{1}{5}\cdot\frac{1}{5}\left[(3)^5 - (1)^5\right]$

$\boxed{= \frac{243}{5} \ln 3 - \frac{242}{25}}$ ✓ Correct KRG

Group 3

10 $\int_0^4 \frac{x-1}{x^2-4x-5}\,dx = \int \frac{x-1}{(x-5)(x+1)}\,dx = \int\left(\frac{A}{x-5} + \frac{B}{x+1}\right)dx$

$x-1 = A(x+1) + B(x-5)$ ✓

$x-1 = Ax + A + Bx - 5B$

$1 = A + B$

$-1 = A - 5B$ $\qquad 1 = 2/3 + B$

$5 = 5A + 5B$ $\qquad 1/3 = B$ ✓

$4 = 6A$

$4/6 = A$

$2/3 = A$ ✓

$= \int \frac{2/3}{x-5}\,dx + \int \frac{1/3}{x+1}\,dx = \frac{2}{3}\ln|x-5| + \frac{1}{3}\ln\underbrace{|x+1|}_{\text{abs val}}\Big|_0^4$

$= \left[\frac{2}{3}\ln|x-5| + \frac{1}{3}\ln|x+1|\right]_0^4$ * see below

$= \left[\frac{1}{3}\ln(x-5)^2 + \frac{1}{3}\ln|x+1|\right]_0^4$ ← making it unnecessarily difficult

$= \frac{1}{3}\ln((-1)^2(5)) - \frac{1}{3}\ln((-5)^2(1))$ no -- this is not how you evaluate the integral

$= \frac{1}{3}\ln(x-5)^2 + \frac{1}{3}\ln|x+1|\Big]_0^4$

$= \frac{1}{3}\ln((-1)^2(5)) - \frac{1}{3}\ln((-5)^2(1))$

same as previous 2 lines no

$= \frac{1}{3}\ln 5 - \frac{1}{3}\ln 25$

$= \frac{1}{3}\ln\frac{5}{25}$

$= \boxed{\frac{1}{3}\ln\frac{1}{5}}$

no

* $\frac{2}{3}\ln|4-5| + \frac{1}{3}\ln|4+1|$

$- \left(\frac{2}{3}\ln|0-5| + \frac{1}{3}\ln|0+1|\right)$

$= \frac{2}{3}\underbrace{\ln 1}_{0} + \frac{1}{3}\ln 5$

$- \frac{2}{3}\ln 5 + \frac{1}{3}\underbrace{\ln 1}_{0}$

$= \boxed{-\frac{1}{3}\ln 5}$ LSG

Group 3

(11) $\displaystyle\int \frac{x-1}{x^2-4x+5}\,dx$ comp. square, $x^2-4x+5 = (x-2)^2+1$

$u = x-2$

$du = dx$

$$= \int \frac{u+1}{u^2+1}\,du =$$

$$\frac{1}{2}\int \frac{2u}{u^2+1}\,du + \int \frac{1}{u^2+1^2}\,du =$$

$$\frac{1}{2}\ln(u^2+1) + \left(\frac{1}{1}\tan^{-1}u\right) + C$$

$$= \frac{1}{2}\ln(x^2-4x+5) + \tan^{-1}(x-2) + C$$

$\int x^4+x^2+1$ $\quad (\ \ \frac{}{2})^2+\frac{}{4}$ Group 3

$$u = x^2+\tfrac{1}{2}$$
$$du = 2x\,dx$$

12 $$\frac{1}{2}\int \frac{du}{u^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{2}\left[\frac{1}{\frac{\sqrt{3}}{2}}\right]\tan^{-1}\left(\frac{u}{\frac{\sqrt{3}}{2}}\right)+C$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2}{\sqrt{3}}\cdot\left(x^2+\tfrac{1}{2}\right)\right)+C$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2}{\sqrt{3}}x^2+\frac{1}{\sqrt{3}}\right)+C$$

✓

Group 3

37. $\int_0^{\frac{\pi}{4}} \tan^3\theta \sec^2\theta \, d\theta$

$x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$\theta\ \cancel{x} = 0\ ;\ x\ \cancel{\theta} = 0$

$\theta\ \cancel{x} = \frac{\pi}{4}\ ;\ x\ \cancel{\theta} = 1$

(you have θ + x backwards)

$= \int_0^1 x^3 \, dx = \frac{1}{4} x^4 \Big|_0^1$

$\frac{1}{4}(1)^4 - \frac{1}{4}(0)^4 = \boxed{\frac{1}{4}}$ ✓

LSG

Group 3

38 $\displaystyle\int_{\pi/6}^{\pi/3} \frac{\sin\theta \cot\theta}{\sec\theta}\, d\theta = \int_{\pi/6}^{\pi/3} \cancel{\sin\theta} \cos\theta \cdot \frac{\cos\theta}{\cancel{\sin\theta}}\, d\theta$ ✓

$\displaystyle = \int_{\pi/6}^{\pi/3} \cos^2\theta \, d\theta$ ✓ $\displaystyle = \int_{\pi/6}^{\pi/3} \left[\frac{1 + \cos 2\theta}{2}\right] d\theta$ ✓

$\displaystyle = \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_{\pi/6}^{\pi/3}$ ✓

forgotten

$2\pi/6 = \frac{\pi}{3}$

$\displaystyle = \frac{1}{2}\left[\left(\frac{\pi}{3} - \frac{\pi}{6}\right) + \underbrace{\left(\sin\frac{2\pi}{3} - \sin\frac{2\pi}{6}\right)}_{\text{no}}\right]$

$\displaystyle = \frac{1}{2} \cdot \frac{\pi}{6} + \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)$

$\displaystyle = \boxed{\frac{\pi}{12}}$ ✓

$\displaystyle \hookrightarrow \frac{1}{2}\left(\sin\frac{2\pi}{3} - \sin\frac{\pi}{3}\right)$

$\displaystyle = \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)$

$\displaystyle = \frac{1}{2}(0) = 0$

$$\int \frac{}{\sec^2\theta - \sec\theta}\, d\theta \qquad u = \sec\theta \qquad du = \sec\theta\tan\theta\, d\theta$$

Group 3

$$\int \frac{du}{u^2 - u} = \int \frac{du}{u(u-1)} \quad \left\{ \frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1} \right.$$

$$1 = Au - A + Bu$$

$$B = 1, \; A = -1$$

$$0 = A + B$$

$$1 = -A$$

$$-\int \frac{1}{\sec\theta} + \int \frac{1}{\sec\theta - 1}$$

$$= -\ln|\sec\theta| + \ln|\sec\theta - 1| + C$$

$$\int -\frac{1}{u}\, du + \int \frac{1}{u-1}\, du$$

$$= -\ln|u| + \ln|u-1| + C$$

Group 3

40. $\displaystyle\int \frac{1}{\sqrt{4y^2-4y-3}}\,dy = \int \frac{1}{\sqrt{4y^2-4y+1-1-3}}\,dy$

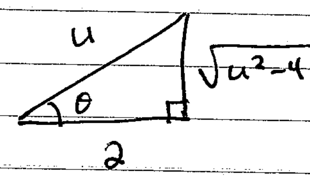
$\displaystyle = \int \frac{1}{\sqrt{(2y-1)^2-4}}\,dy$ $\qquad u = 2y-1$

$\qquad du = 2\,dy$

$\displaystyle = \frac{1}{2}\int \frac{1}{\sqrt{u^2-2^2}}\,du = \frac{1}{2}\ln\left|u+\sqrt{u^2-4}\right| + C$ Warning: on the test, you will either need to know this formula or be able to derive it using trig substitution.

$\boxed{\displaystyle = \frac{1}{2}\ln\left|2y-1+\sqrt{4y^2-4y-3}\right| + C}$

✓



Group 5

$$\int \frac{\sin 2x}{1+\cos^4 x}\,dx$$

$u = \cos^2 x$

$du = -\sin 2x\,dx$

$\hookrightarrow$ details (show work)

$u = \cos^2 x = \dfrac{1+\cos 2x}{2}$

$\therefore du = -\sin 2x\,dx$

(65)

$$= -\int \frac{1}{u^2+1}\,du = -\tan^{-1} u + C$$

$$= -\tan^{-1}(\cos^2 x) + C$$

✓

Group 3

66. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\ln(\tan x)}{\sin x \cos x}\, dx$

$u = \ln(\tan x)$

$du = \frac{1}{\tan x} \sec^2 x \, dx$

$du = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} \, dx$

$du = \frac{1}{\sin x \cos x} \, dx$

$x = \frac{\pi}{4}$; $\underbrace{u = 0}_{\ln 1 = 0}$

$x = \frac{\pi}{3}$; $u = \ln\sqrt{3}$

$= \displaystyle\int_0^{\ln\sqrt{3}} u \cdot du$

$= \frac{1}{2} u^2 \Big|_0^{\ln\sqrt{3}}$

$= \frac{1}{2}(\ln\sqrt{3})^2 - \frac{1}{2} \cancel{\ln(0)}^{\,0}\,{}^2$

not $\ln(0)$ but 0

does not exist

$= \boxed{\frac{1}{2}(\ln\sqrt{3})^2}$

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx \cdot \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}} =$$ Group 3

67

$$\int\left(\sqrt{x+1}-\sqrt{x}\right)dx$$

$$= \int (x+1)^{1/2} - (x)^{1/2}\,dx$$

$$= \frac{2}{3}\left[(x+1)^{3/2} - (x)^{3/2}\right] + C$$

Group 5

68 $\int \frac{x^2}{x^6+3x^3+2}dx = \int \frac{x^2}{x^6+2x^3+x^3+2}dx$

$= \int \frac{x^2}{x^3(x^3+2)+1(x^3+2)}dx = \int \frac{x^2}{(x^3+2)(x^3+1)}dx$ ✓ okay to ~~do~~ this point

$= \boxed{\int \frac{x^2}{x^3+2}dx - \int \frac{x^2}{x^3+1}dx}$ no -- or if true, you must justify it by showing work.

$= \frac{1}{3}\ln|x^3+2| - \frac{1}{3}\ln|x^3+1| + C$ no

$= \boxed{\frac{1}{3}\ln\left|\left(\frac{x^3+2}{x^3+1}\right)\right| + C}$

$\int \frac{x^2}{(x^3+2)(x^3+1)}dx$ let $u = x^3$

then $x^3+2 = u+2$

and $x^3+1 = u+1$

$du = 3x^2\,dx$

So the integral is $\frac{1}{3}\int \frac{1}{(u+2)(u+1)}du$

now use partial fraction decomposition:

$\frac{1}{(u+2)(u+1)} = \frac{A}{u+2} + \frac{B}{u+1} \rightarrow \dots \rightarrow A=1,\quad B=-1$

$\frac{1}{3}\int\left(\frac{1}{u+2} - \frac{1}{u+1}\right)du = \frac{1}{3}\Big[\ln|u+2| - \ln|u+1|\Big] + c$

$= \frac{1}{3}\ln\left|\frac{x^3+2}{x^3+1}\right| + c$