Group 2

5 $\int \frac{t}{t^4+2}\,dx$

$u = t^2$

$du = 2t\,dx$

$\frac{1}{2}\int \frac{1}{u^2+2}\,du$

$= \frac{\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)}{2\sqrt{2}} = \frac{\tan^{-1}\left(\frac{t^2}{\sqrt{2}}\right)}{2\sqrt{2}} + C$ ✓

correct KRG

Khalid Hakeem

Q6) let $u = 2x+1$

$$\Rightarrow x = \frac{u-1}{2}$$

$$dx = \frac{1}{2} du$$

$$\therefore \int \frac{x}{(2x+1)^3} dx = \int \frac{\frac{1}{2}(u-1)}{u^3} \frac{1}{2} du = \int \frac{1}{4}(u^{-2} - u^{-3}) du$$

$$= \frac{1}{4}\left(-u^{-1} - \frac{1}{-2}u^{-2}\right) + C = -\frac{1}{4u} + \frac{1}{8u^2} + C$$

$$= -\frac{1}{4(2x+1)} + \frac{1}{8(2x+1)^2} + C.$$

To finish

$$\int_0^1 \frac{x}{(2x+1)^3} dx = -\frac{1}{4(2x+1)} + \frac{1}{8(2x+1)^2}\Bigg|_0^1$$

$$= -\frac{1}{4(3)} + \frac{1}{8(3^2)} - \left(-\frac{1}{4} + \frac{1}{8}\right)$$

$$= \frac{1}{8}\left[-\frac{2}{3} + \frac{1}{3^2} + 2 - 1\right]$$

$$= \frac{1}{8}\left[\frac{-6+1+9}{3^2}\right]$$

$$= \frac{1}{8}\left[\frac{4}{3^2}\right] = \boxed{\frac{1}{18}}$$ Answer

Grap 2

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#7. $\int_{-1}^{1} \frac{e^{\arctan y}}{1+y^2}\,dy \rightarrow$ let $u = \arctan y$, $du = \frac{1}{1+y^2}\,dy$

$$\int_{-1}^{1} e^u\,du$$

$$\left[e^u du\right]_{-1}^{1} \rightarrow \left[e^{\arctan y}\right]_{-1}^{1} \rightarrow e^{\arctan(-1)} + e^{\arctan(1)}$$

$$\boxed{e^{\frac{\pi}{4}} - e^{-\frac{\pi}{4}}}$$

✓

only error
KRB

[redacted] - Group 2

#8 $\int t \sin t \cos t \, dt$ $\qquad \sin t \cos t = \frac{1}{2}\sin(2t)$

$\frac{1}{2}\int t\sin(2t)$

sub 1

$u = t \qquad \cdot\, dv = \sin 2t$

$du = dt \qquad v = -\frac{1}{2}\cos 2t$

$uv - \int v\,du$

$\frac{1}{2}\left[t\left(-\frac{1}{2}\cos 2t\right) + \frac{1}{2}\int (\cos 2t)(dt)\right]$

$-\frac{1}{4}t\cos(2t) + \frac{1}{4}\int \cos(2t)\,dt$

sub 2

$u = 2t$

$du = 2\,dt$

$\frac{1}{2}du = dt$

$-\frac{1}{4}t\cos(2t) + \left[\frac{1}{4}\left(\frac{1}{2}\right)\int \cos(u)\,du\right]$

$-\frac{1}{4}t\cos(2t) + \frac{1}{8}\int \cos(u)\,du$

$-\frac{1}{4}t\cos(2t) + \frac{1}{8}\sin(u) =$

$\boxed{\frac{1}{8}\sin(2t) - \frac{1}{4}t\cos(2t)} + C$

✓

Don't forget "C"

KRG

33 $\int \sqrt{3-2x-x^2}\,dx$

$\int \sqrt{4-(x+1)^2}\,dx$

$u = x+1$

$du = 1dx$

$\int \sqrt{4-u^2}\,du$

$u = 2\sin\theta$

$\int \sqrt{4-4\sin^2\theta} = \int 4\cos^2\theta$

$4\int \left(\frac{1}{2}\cos(2\theta)+\frac{1}{2}\right) d\theta$

$4\int \frac{1}{2} + 2\int \cos 2\theta$

$4\left(\frac{1}{2}\theta\right) + 2\int \cos 2\theta$] X why

$2x + 2\int 2\left(\frac{\sqrt{4-u^2}}{2}\right)$

$2x + 2\int \sqrt{4-u^2}$

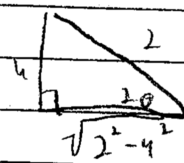
$2x + 2\int (4-u^2)^{1/2} = 2x + 2\left[\frac{3}{2}(4-u^2)^{3/2}(-2u)\right] = 2x+2\left[\frac{3}{2}(4-u^2)^{3/2}(-2u)\right]$

$2x + 3\left(4-(x+1)^2\right)^{3/2}(-2(x+1))$

$2x + 3\left(4-(x+1)^2\right)^{3/2}(-2x-2) + C$ X

$\frac{u}{2} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{u}{2}\right)$



$y = 2\sin\theta$

$dy = 2\cos\theta\,d\theta$

$4\int \frac{1}{2}d\theta + 2\int \cos 2\theta\,d\theta$

$2\theta + \sin 2\theta + C$

$2\theta + 2\sin\theta\cos\theta + C$

$2\arcsin\left(\frac{x+1}{2}\right) + 2\,\frac{x+1}{2}\cdot\frac{\sqrt{2-(x+1)^2}}{2} + C$

$$\boxed{2\arcsin\left(\frac{x+1}{2}\right) + \frac{(x+1)}{2}\sqrt{3-2x-x^2} + C}$$

answer

KRG

Q34) $\displaystyle\int_{\pi/4}^{\pi/2} \frac{1+4\cot x}{4-\cot x}\,dx$

$\displaystyle\int_{\pi/4}^{\pi/2} \frac{(1+4\cos x/\sin x)}{(4-\cancel{\cos x/\sin x}\ \cancel{\sin x/\cos x})} \cdot \frac{\sin x}{\sin x}\,dx$ X

$\displaystyle\int_{\pi/4}^{\pi/2} \frac{\sin x + 4\cos x}{4\sin x - \cos x}\,dx$ ✓

where did these limits come from?

$\displaystyle\int_{3/\sqrt{2}}^{4} \frac{1}{u}\,du$ ✓ where $u = 4\sin x - \cos x$
$du = (4\cos x + \sin x)\,dx$

$\left[\ln|u|\right]_{3/\sqrt{2}}^{4}$

$\ln 4 - \ln \dfrac{3}{\sqrt{2}}$

$\ln\left(\dfrac{4\sqrt{2}}{3}\right)$ ✓

correct
KRG

Grap 2

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#35 $\int \cos 2x \cos 6x \, dx$

$\int \frac{1}{2}[\cos(-4x) + \cos(8x)] + C$

$\frac{1}{2}\left[-\frac{1}{4}\sin -4x + \frac{1}{8}\sin 8x\right]$

$-\frac{1}{8}\sin(-4x) + \frac{1}{16}\sin 8x + C$ ✓

which is also equal to $\frac{1}{8}\sin 4x + \frac{1}{16}\sin 8x + C$

OK

KRG

[redacted] Group 2

#36 $\displaystyle\int_{-\frac{\pi}{4}}^{\pi/4} \frac{x^2 \tan x}{1+\cos^4 x}\,dx \qquad f(-x) = -f(x)$

$$f(x) = \frac{x^2 \tan x}{1+\cos^4 x}$$

$f(-x) = (-x)^2 = x^2 \longrightarrow$ even

$\tan(-x) = -\tan x \longrightarrow$ odd

$1+(\cos(-x))^4 = 1+(\cos x)^4 \rightarrow$ even

$$f(-x) = \frac{x^2(-\tan x)}{1+\cos^4 x} = \frac{-x^2 \tan x}{1+\cos^4 x} = -f(x)$$

so $f(x)$ is odd

and $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$ for odd functions.

$$\int_{-\pi/4}^{0} \frac{x^2 \tan x}{1+\cos^4 x}\,dx + \int_{0}^{\pi/4} \frac{x^2 \tan x}{1+\cos^4 x}\,dx = \boxed{0}$$

this is not necessary [redacted]

All odd functions defined on $[-a,a]$ have the property that $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$.

61) $\int \frac{d\theta}{\cos\theta+1} = \int \frac{1}{\sin^2\theta} - \frac{\cos\theta}{\sin^2\theta} = \int \csc^2\theta - \cot\theta\csc\theta$

$\int \csc^2\theta = -\cot\theta$

$\int \csc\theta\cot\theta = -\csc\theta$

$-\cot\theta + \csc\theta + C$ ✓ OK RG

Explain how

$$\frac{1}{\cos\theta+1} = \frac{1}{\sin^2\theta} - \frac{\cos\theta}{\sin^2\theta}.$$

(i.e. $\frac{1}{\cos\theta+1} = \frac{1}{\cos\theta+1}\,\frac{\cos\theta-1}{\cos\theta-1}$

$= \frac{1}{\cos^2\theta-1}\,\cos\theta-1$

$= -\frac{1}{\sin^2\theta}(\cos\theta-1)$

$= \frac{1}{\sin^2\theta} - \frac{\cos\theta}{\sin^2\theta}$)

Group 2

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63. $\int \sqrt{x}\, e^{\sqrt{x}}\, dx$

$y = \sqrt{x}$

$\frac{dy}{dx} = \cancel{x^{\frac{1}{2}}}\ \frac{1}{2} x^{-\frac{1}{2}}$

$\int y e^{y} dx$

$dy = \frac{1}{2} x^{-\frac{1}{2}} dx$ ✓

$2\sqrt{x}\, dy = dx$

$2y\, dy = dx$ ✓

$2\int y^2 e^y dy$

$uv - \int v du$

$2\int u^2 e^u du$

$u = y^2 \qquad v = e^y dy$

$du = 2y dy \qquad v = e^y$

$2\left[y^2 e^y - \int e^y 2y\, dy\right]$

$2y^2 e^y - 4\int e^y y\, dy \qquad w = y \quad dw = dy \quad V = e^y$

$2y^2 e^y - 4\left[y e^y - \int e^y dy\right.$

$2y^2 e^y - 4y e^y + 4e^y + C$

$2x e^{\sqrt{x}} - 4\sqrt{x} e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$

$2e^{\sqrt{x}}\left[x - 2\sqrt{x} + 2\right] + C$ ✓

fine KRG

Group 2

#64 $\displaystyle\int \frac{1}{\sqrt{\sqrt{x}+1}}\,dx$ $\boxed{\text{sub 1}}$ $u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$ $\cap$ $2u\,du = dx$

$2\displaystyle\int \frac{u}{\sqrt{u+1}}\,du$ $\boxed{\text{sub 2}}$ $w = u+1$ $\quad 1 =$

$dw = du$

$2\displaystyle\int \frac{w-1}{\sqrt{w}}\,dw = 2\int\left(\sqrt{w} - \frac{1}{\sqrt{w}}\right)dw$

$2\displaystyle\int \sqrt{w}\,dw - 2\int \frac{1}{\sqrt{w}}\,dw$ $\quad w^{1/2}$

$2\left(\frac{2}{3}\right)w^{3/2} - 2\left(2\sqrt{w}\right) + C$

$\boxed{\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2} - 4\sqrt{\sqrt{x}+1} + C}$ ✓

fine as is

$v = \sqrt{x} + 1$ also works

$x = (v-1)^2 \quad dx = 2(v-1)\,dv$

$\displaystyle\int \frac{1}{\sqrt{\sqrt{x}+1}}\,dx = \int \frac{1}{\sqrt{v}}\,2(v-1)\,dv$

$= 2\displaystyle\int\left(v^{\frac{1}{2}} - v^{-\frac{1}{2}}\right)dv$

$= 2\left[\frac{v^{3/2}}{\frac{3}{2}} - \frac{v^{\frac{1}{2}}}{\frac{1}{2}}\right] + C$

$= \frac{4}{3}\left(\sqrt{x}+1\right)^{3/2} - 4\sqrt{\sqrt{x}+1} + C$