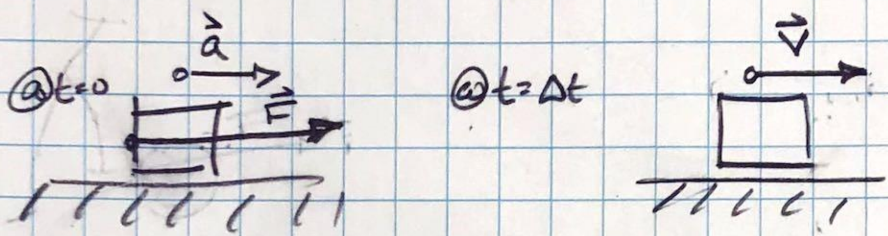
# CH 9: Momentum

## Impulse (J)

When Force (F) is applied over a specified duration ($\Delta t$):



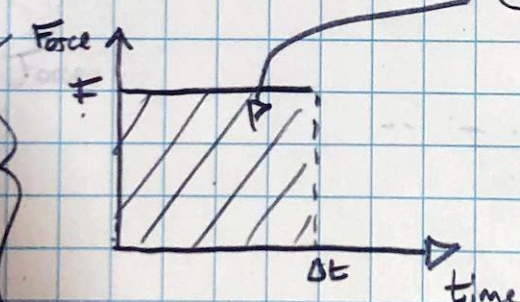

$$\left\{ \vec{F} = m\vec{a} = m\frac{\Delta \vec{v}}{\Delta t} \right\} \cdot \Delta t$$

$$\vec{F} \cdot \Delta t = m \cdot \Delta \vec{v}$$

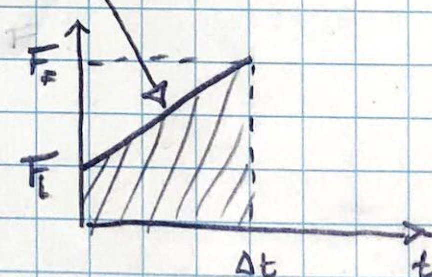
Impulse: $\vec{J} = \vec{F} \cdot \Delta t \equiv$ Change in momentum: $\vec{p} = m\vec{v}$

Also equal to $\boxed{\text{area}}$ under (force, time) curve

$F > 0$

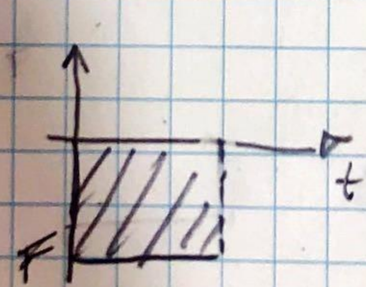



$J = F\Delta t$

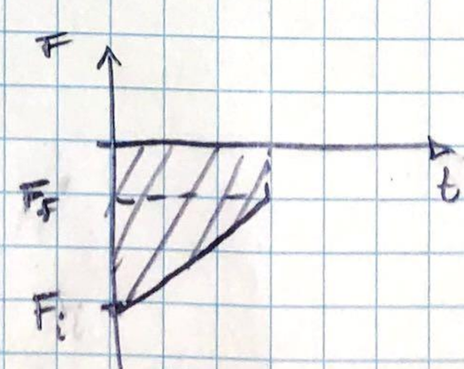



$J = \frac{1}{2}(F_i + F_f)\Delta t$

$F < 0$




$J < 0$

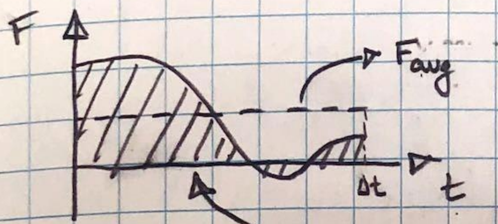



$J < 0$

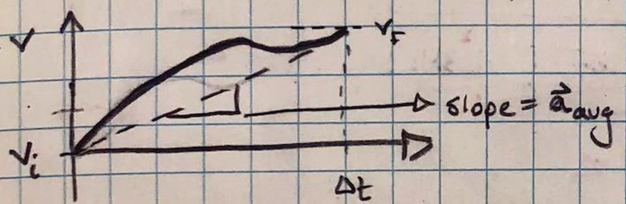
**Momentum:** product of inertia and velocity

$$\boxed{\vec{p} = m\vec{v}}$$

Impulses cause changes in Momentum




$J = \text{Area} = F_{avg}\,\Delta t$




$$a_{avg} = \frac{v_F - v_i}{\Delta t}$$

Newton's second law:

$$F_{avg} = m\,a_{avg} = m\,\frac{v_F - v_i}{\Delta t}$$

$$\vec{F}_{avg}\,\Delta t = \boxed{m\,\Delta\vec{v}}$$

$\Delta\vec{p}$ = change in momentum

$$\therefore\ \boxed{\vec{J} = \Delta\vec{p}}$$

$$\boxed{p_F = p_i + J}$$

Momentum is a vector + obeys addition/subtraction rules:

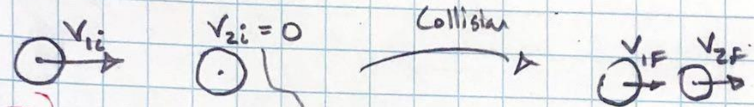
For a collection of objects

$$\vec{p}_{net} = \vec{p}_1 + \vec{p}_2 + \dots = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i$$

★ Net Momentum ($\vec{p}_{net}$) is conserved unless acted on by an external impulse

→ Newton's 1st + 2nd Laws

Collision: $\boxed{\vec{P}_i = \vec{P}_F}$



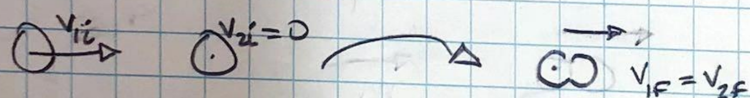

$$\vec{P}_i = \underbrace{m_1 v_1}_{P_{1i}} + \underbrace{m_2 v_2}_{P_{2i}} = m_1 v_1 \qquad \vec{P}_F = \underbrace{m_1 v_{1F}}_{P_{1F}} + \underbrace{m_2 v_{2F}}_{P_{2F}}$$

$$\Delta \vec{P} = 0 \Rightarrow m_2 v_{2F} = m_1 (v_{1i} - v_{1F})$$

$$\boxed{\Delta \vec{p}_2 = -\Delta \vec{p}_1}$$

~~IF $v_{1F} = v_{2i}$, collision is perfectly elastic!~~

IF $\boxed{v_{1F} = v_{2F}}$, collision is perfectly inelastic!



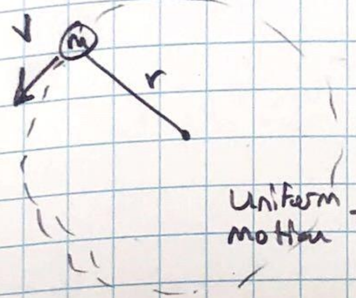

$$\vec{P}_i = m_1 v_{1i} \qquad \vec{P}_F = m_1 v_{1F} + m_2 v_{2F} = (m_1 + m_2) v_F$$

Perfectly Inelastic

$$\boxed{v_f = \frac{m_1 v_{1i}}{(m_1 + m_2)}}$$

## Angular Momentum (L)

↳ Momentum of object in circular motion.




Uniform motion $\Rightarrow$

particle speed $v_t = \omega r$ (angular velocity: $\omega$; radius: $r$)

particle centrepital acceleration $a_c = \frac{v^2}{r}$

Tangential forces induce tangential acceleration

$$F_t = m\,a_t = m\frac{\Delta V_t}{\Delta t}$$

$$\underbrace{F_t\,\Delta t}_{\text{tangetiall Impulse}} = \underbrace{m\,\Delta V_t}_{\text{tangential momentum}}$$

angular acceleration $= \frac{\Delta\omega}{\Delta t}$

Recall: $F_t = \frac{\tau}{r} = \frac{\text{torque}}{\text{radius}}$ & $a_t = \alpha r$

$$\Rightarrow \quad \tau = \boxed{mr^2}\,\alpha = I\,\frac{\Delta\omega}{\Delta t}$$

or

$$\boxed{\tau\,\Delta t} = \boxed{I\,\Delta\omega}$$

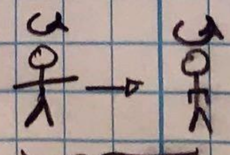
Impulsive torque causes change in angular momentum

Angular Momentum : $\boxed{L = I\omega}$

Changes can be from $\Delta\omega$ or $\Delta I$!

$$\Delta L = \underbrace{I\,\Delta\omega + \Delta I\,\omega}$$

e.g.



decreased moment of Inertia