CH6: Circular Motion, Orbits & Gravity!

- Recall: In uniform circular motion, acceleration ($\vec{a}_c$) points toward the center of the circle...

$$\boxed{\text{Centrepital acceleration: } a_c = \frac{v^2}{r}}$$

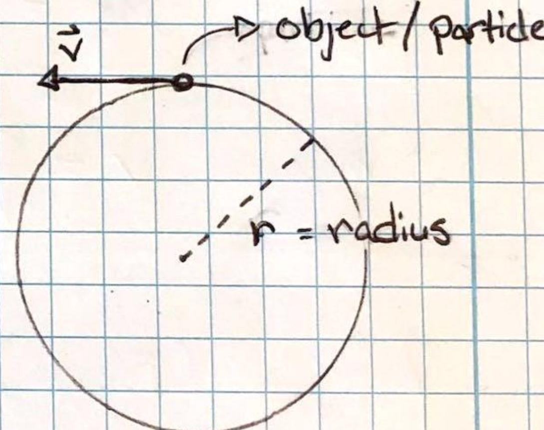
magnitude of $\vec{a}_c$

Based on Newton's 2nd Law ($\vec{F} = m\vec{a}$), there must be a force pointing toward the center of the circle:

Centrepital force: $F_c = ma_c = m\frac{v^2}{r}$

magnitude of $\vec{F}_c$

Kinematics of Circular motion:



τ = period = time to go around circle once.

Circumference $= 2\pi r$

Speed $= v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{\tau}$

$$\therefore \boxed{\tau = \frac{2\pi r}{v}}$$

Frequency $(f) = \frac{\text{\# of Revolutions}}{\text{time}} = \frac{1}{\tau}$

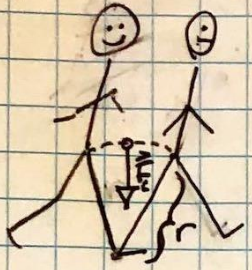
We can re-write a_c in terms of τ + f:

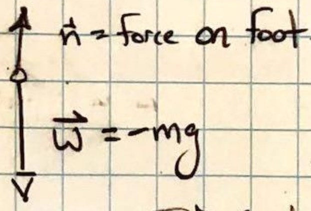
$$a_c = \frac{v^2}{r} = \frac{(2\pi r/\tau)^2}{r} = \left(\frac{2\pi}{\tau}\right)^2 r$$

or $f = \frac{1}{\tau}$

$$a_c = (2\pi f)^2 r$$

Currious case of Walking Motion:





center of circle is down!

$$F_c = \vec{n} + \vec{w} = m\vec{a}_c = -m\frac{v^2}{r}$$

Maximum force F_c is when $\vec{n} = 0$ (just before "running")

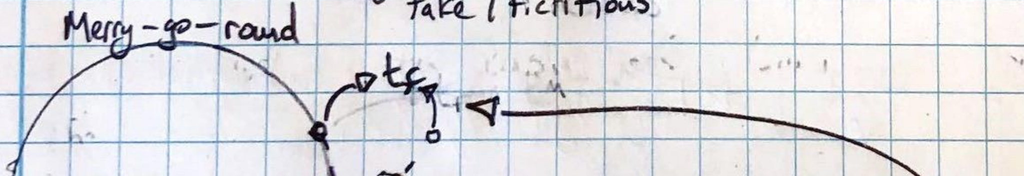
$$F_{c,max} = -\cancel{m}g = -\cancel{m}\frac{v^2}{r}$$

maximum $v = \sqrt{gr}$ → tall people walk faster.

Apparent (Pseudo) Forces:

↳ fake / Fictitious

Merry-go-round



you have velocity $\vec{v}$, some time later...

IF you stay on the ride you follow circle...

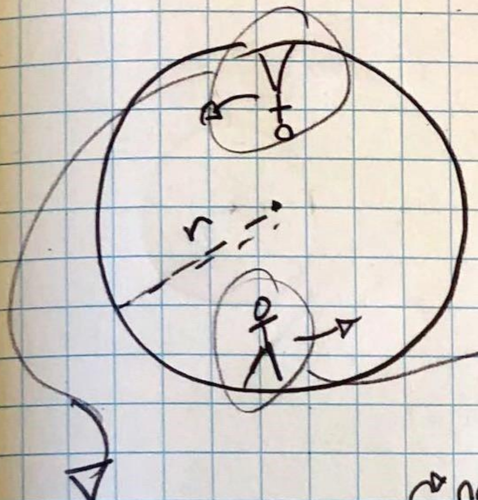
IF you let go you'll fly off in a straight line away from ride...

- From your perspective there was something "pulling" you away from the ride:

Centri$\boxed{\text{fugal}}$ Force = fake force!

Apparent weight: Similar to elevator...

you "feel" the normal force $\vec{n}$.



y = up

$$F_{net,y} = \vec{n} + \vec{w} = n - mg = ma_c = m\frac{v^2}{r}$$

$$n = mg + m\frac{v^2}{r}$$

normal force points down

$$F_{net,y} = \vec{n} + \vec{w} = -n - mg = -ma_c = -m\frac{v^2}{r}$$

Apparent Weight $\boxed{w_{app} = m\frac{v^2}{r} - mg}$

∴ apparent weight is smaller @ top than @ bottom!

if $n = 0$, you'll "feel" weightless... and might fall!

$$n = 0 = m\frac{v^2}{r} - mg = w_{app}$$

$$\cancel{m}\frac{v^2}{r} = \cancel{m}g$$

$$v = \sqrt{gr}$$ critical speed, below which you fall

this is the exact speed of sattelites... they don't fall to earth!

Newton's Law of Gravity:

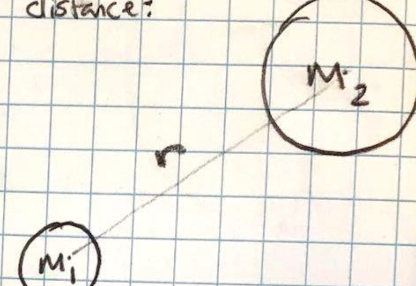
Gravity is universal!

↳ all objects experience Gravity.

↳ each object attracts all other objects

1) Force of gravity weakens w/ distance:

$$F_g \propto \frac{1}{r^2}$$



2) Force of gravity increases w/ mass of both objects:

$$F_g \propto M_1 M_2$$

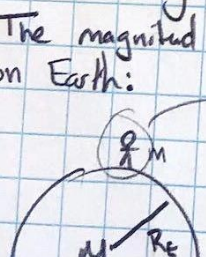
Combining 1 & 2:

$$F_g = G \frac{M_1 M_2}{r^2}$$

↳ gravitational constant: $G = 6.67 \times 10^{-11}\ N\, m^2/Kg^2$

The magnitud of (g) on the surface of a planet:

on Earth:

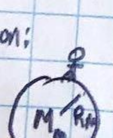


$$\vec{w} = -mg = -G\frac{m M_E}{R_E^2}$$

$\therefore$ on Earth: $g = G\frac{M_E}{R_E^2}$

$$\boxed{g = G\frac{M_{Planet}}{R_{Planet}^2}}$$

on Moon:



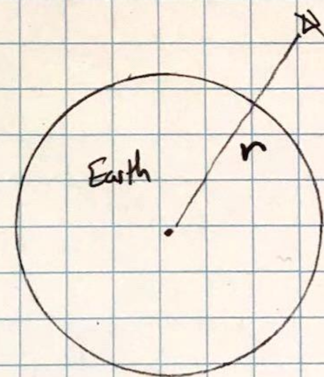
$$\vec{w} = -mg_{moon} = -G\frac{m M_M}{R_M^2}$$

$\therefore$ on Moon: $g = G\frac{M_M}{R_M^2}$

Orbital Speed: speed of a sattelite

Recall:

A sattelite does not fall to earth because its apparent weigt is zero:



$$W_{app} = \frac{m v^2}{r} - mg = 0$$

$$\therefore \; v = \sqrt{gr}$$

$$= \sqrt{\frac{GM_E}{r^2} r} = \sqrt{\frac{GM_E}{r}}$$

A sattelite circles the Earth w/ period (τ)

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{\tau} \Rightarrow \tau = 2\pi \frac{r^{3/2}}{\sqrt{GM_E}}$$