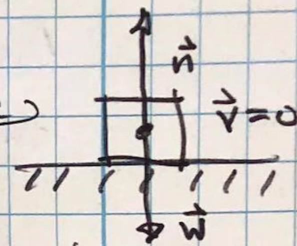
CH 5: Applying Newton's Laws



$a = 0$

Static Equilibrium: object at rest ... no net force $\rightarrow \vec{F}_{net} = 0$

Stationary

Dynamic Equilibrium: uniform 1D motion ... $\vec{v}$ = constant

$$\boxed{\vec{F}_{net} = 0}$$

Recall:

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \ldots = 0 \quad \text{Equilibrium}$$

components

$$\vec{F}_{x\,net} = \vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_{3x} \ldots = 0$$

$$\vec{F}_{y,net} = \vec{F}_{1y} + \vec{F}_{2y} + \ldots = 0$$

Since $\vec{F}_{net} = 0$, $\vec{a} = 0$

Newton's 2nd Law:

$$\boxed{\vec{F}_{net} = \overset{\text{mass}}{m}\,\vec{a}} \qquad \text{units: } \left[kg \frac{m}{s^2} \right]$$

---

if $\vec{F}_{net} \neq 0$, the object accelerates $(\vec{a} \neq 0)$

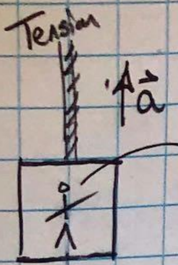
$$\left.\begin{aligned} \vec{F}_{net,x} &= m \cdot \vec{a}_x \\ \vec{F}_{net,y} &= m \cdot \vec{a}_y \end{aligned}\right\} \text{2nd Law in Component form.}$$

Free-Fall: the weight of an object is the $\overbrace{\text{force of gravity}}^{\vec{F}_g}$.

$$\vec{F}_g = \vec{w} = -mg \;\; (\text{mass} \cdot \text{grav. acc, points down}) \qquad \text{or } |\vec{w}|$$

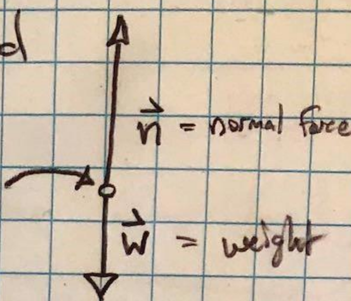
$$\boxed{\therefore \quad a_y = \frac{\vec{w}}{m} = -g} \qquad \text{mass} = \frac{\text{weight}}{9.8\,m/s^2}$$

Apparent Weight:



Elevator accelerating upward

Free body diagram:



$|\vec{n}| > |\vec{w}|$

Newton's 2nd Law $\left\{ \vec{F}_{net} = \vec{F}_y = n_y + w_y = \underbrace{n_y - w = m a_y}_{} \right.$ (normal force → n_y; weight → w)

$F_x = 0 = m a_x$

$n_y = w + m a_y$ ✓

you feel the normal force. IF $a_y > 0$ (going up) you feel heavier

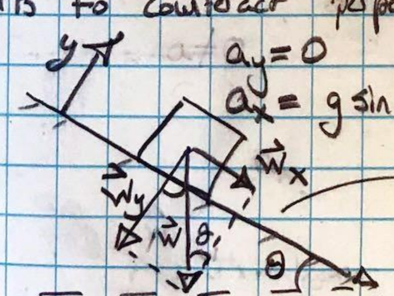
IF $a_y < 0$ you feel lighter... if $a_y = -g$, you feel weightless!

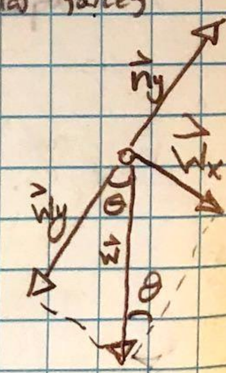
Normal Force:

Force that surface exerts to counteract perpendicular forces

$\vec{a} = 0$

$F_{net} = \vec{w} + \vec{n} = 0$





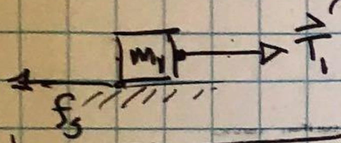
$F_{net,y} = \vec{n}_y + \vec{w}_y = 0$

$\vec{w}_y = -mg\cos\theta = -\vec{n}_y$

$w_x = \cancel{m} g \sin\theta = \cancel{m} \vec{a}_x$

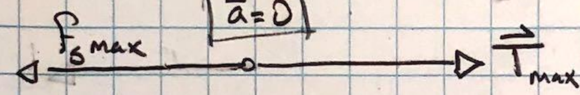
$\therefore \vec{a}_y = 0$ & $a_x = g \sin\theta$

Static Friction



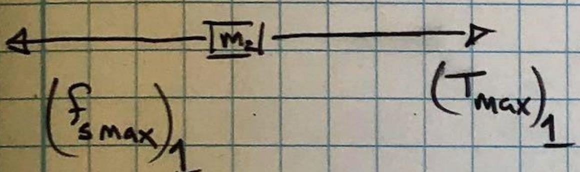
$\rightarrow$ Increase $|\vec{T}|$ until block almost moves

$\boxed{\vec{a} = 0}$



$\rightarrow$ Static equilibrium $\vec{f}_s = -\vec{T}$

Once you know $\vec{T}_{max}$ double the mass + re-do... $\rightarrow m_2 = 2m_1$



... but block doesn't budge.

increase $|\vec{T}|$ to $(T_{max})_2$ ...

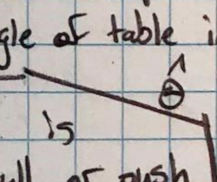
result $\Rightarrow$ $(T_{max})_2 = 2\,(T_{max})_1$

$$\therefore \quad f_s \propto \text{mass}! \qquad \boxed{\frac{(T_{max})_2}{m_2} = \frac{(T_{max})_1}{m_1}}$$

$\rightarrow$ actually its directly proportional to the normal force!

$$f_s = \underbrace{\mu_s}_{\text{Coefficient of Static Friction}} \cdot \underbrace{\vec{n}}_{\text{normal force}}$$

$\rightarrow$ Conduct same experiment but vary angle of table instead of (m).



★ direction of f_s is opposite the pull or push

Kinetic Friction: similarly,

$$f_k = \mu_k \cdot \vec{n}$$

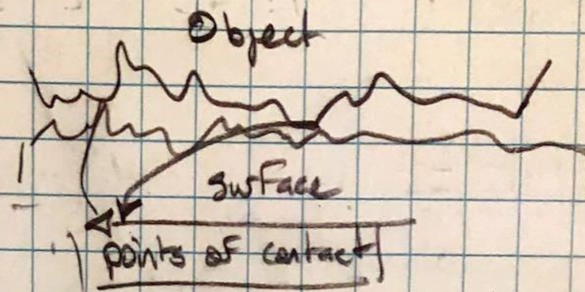
$\rightarrow$ direction is opposite $\vec{v}$!

but $\mu_k \neq \mu_s$ for almost all materials

Rolling Friction: can be written as $f_r = \mu_r \vec{n}$

Friction:

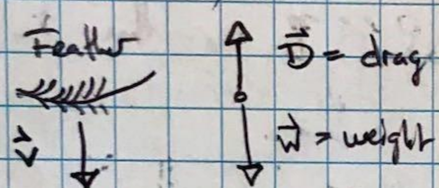
1) Microscopic roughness :



2) contact depends on how hard object / surface are pushed together, i.e., depends on normal force $\vec{n}$

Drag:

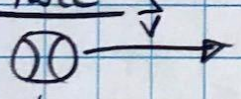
(or air resistance)



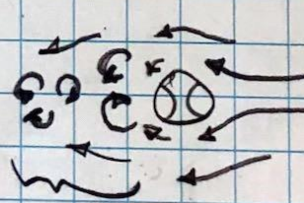
1) opposes velocity : direction $(\vec{D})$ = − direction $(\vec{v})$

2) increases as $|\vec{v}|$ increases !

Inertial Force



air relative to ball →



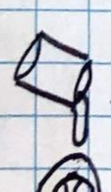
$f_I \sim \rho v^2 L^2$

ρ → density of air
v^2 → (velocity)^2
L^2 → cross-section area

wake slows ball down!
→ large scale
→ small scale

Viscous Force:

honey



→ friction btw molecules resist motion

$f_v \sim \eta v L$

η → viscosity
v → velocity
L → Diameter / circumference

Reynolds #: relative size of Inertial to Viscous Force

$$Re = \frac{f_I}{f_v} \sim \frac{\rho v^2 L^2}{\eta L v} = \frac{\rho \cdot vL}{\eta} \quad \left(\text{no units: } \frac{N}{N}\right)$$

much gtr than

if $(vL) \sim 1 \frac{m^2}{s}$ & $\rho \gg \eta$, then f_I dominates

" " + $\rho \ll \eta$, then f_v dominates

In most cases, Re is very large. $\left(f_I \gg f_v\right)$

Drag @ high Re:

$$\boxed{\vec{D} = \frac{1}{2} C_D \rho A v^2}$$

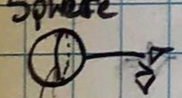
ρ = density of fluid

C_D = drag coefficient depends on aerodynamics

A = cross-sectional area

v = velocity

Sphere

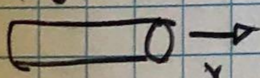


$C_D \sim 0.5$

$A = \pi r^2$

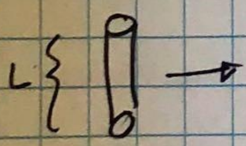
Same A, but different C_D due to shape!

Cylendar



$C_D \sim 0.8$

$A = \pi r^2$

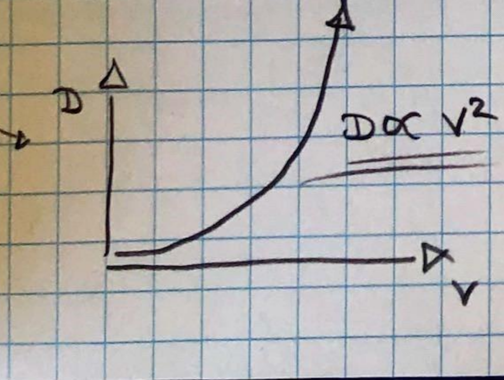


$C_D \sim 1.1$

$A = 2r \cdot L$

larger A and C_D!

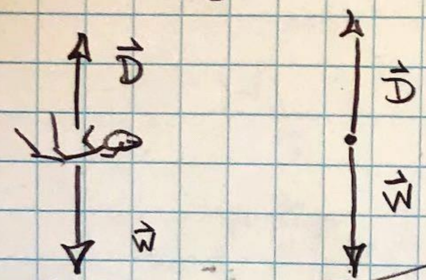
not aerodynamic!



$\vec{D}$ is a quadratic function of $\vec{v}$

at high Re!

Terminal Velocity: max speed of object falling w/ drag



$$\vec{F}_{net} = \vec{D} + \vec{W} = 0$$

Large Re $\Rightarrow$ $\vec{D} = \frac{C_D}{2}\rho \overset{\text{Area}}{A} v^2 = -\vec{W} = mg$

inertial

$$\boxed{v_{max} = \sqrt{2\frac{mg}{C_D \rho A}}}$$

Small Re $\Rightarrow$ $\vec{D} \propto -\overset{\text{viscosity}}{\eta}\overset{\text{velocity}}{\vec{v}}$

Viscous

$$= -b\,\eta\vec{v}$$

b: varies depending on shape of object.

for a sphere:

$$\vec{D} = -(6\pi r)\,\eta\vec{v} \quad (\text{sign of } \vec{D} = \text{opposite } \vec{v}!)$$

$$\therefore \quad \vec{D} = -mg$$

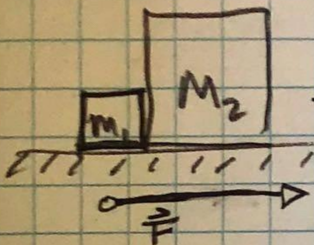
$$-6\pi r\,\eta v = -mg$$

$$\boxed{v_{max} = \left(\frac{mg}{6\pi r\eta}\right)}$$

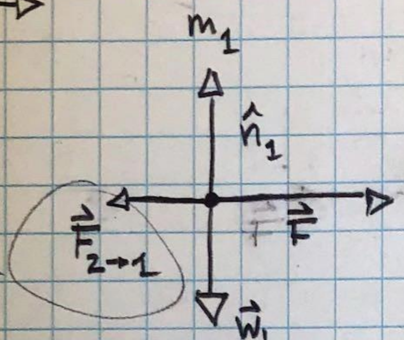
Objects In Contact

Say we push on m_1 to the right w/ Force $\vec{F}$,

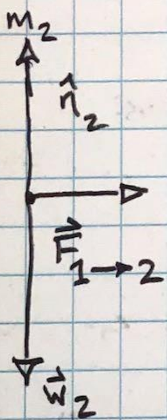
both $m_1 + M_2$ move!



$\Longrightarrow$ decompose objects



Force of mass 2 on mass 1



net force on m_1

$$(F_{2-1} + F) = \vec{F}_{net,1} < \vec{F}_{net,2}$$

smaller than

y-component forces are zero $\vec{F}_{net,y} = 0$ so no acceleration

x-component

total mass $(m_1 + m_2)$

$$\vec{F}_{net} = m_{tot}\,\vec{a}$$

$$\vec{F}_{2\to1} + \vec{F} + \vec{F}_{1\to2} = (m_1 + m_2)\,a$$

(action/reaction pair)

$$\boxed{\vec{F}_{2\to1} = -\vec{F}_{1\to2}} \quad \therefore \quad \boxed{a = \frac{F}{(m_1 + m_2)}}$$

Blocks have same acc: $a_1 = a_2$

$$\vec{F}_{2\to1} + \vec{F} = m_1 a_1 \qquad \overbrace{F_{1\to2} = m_2 a_2}^{M_2}$$

$$a_1 = \frac{\vec{F}_{21} + F}{m_1} = a_2 = \frac{F_{1\to2}}{m_2}$$

$$F_{21} + F = \frac{m_1}{m_2} F_{1\to2}$$

$$\boxed{\therefore\ F_{12} = F/\left(1 + \frac{m_1}{m_2}\right)}$$

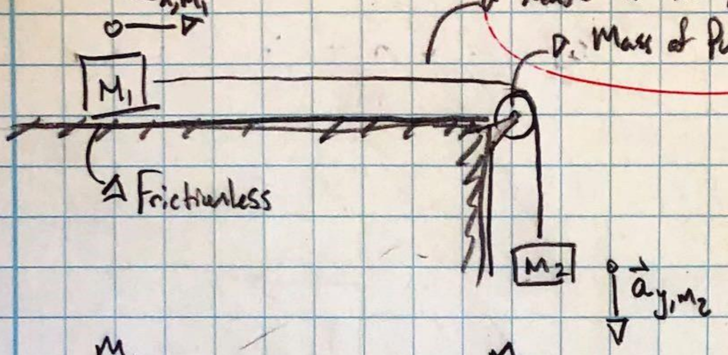
Ropes + Pulleys

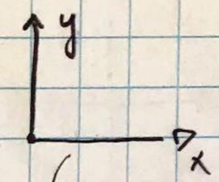
1st assume they have zero mass ⇒ massless string approx

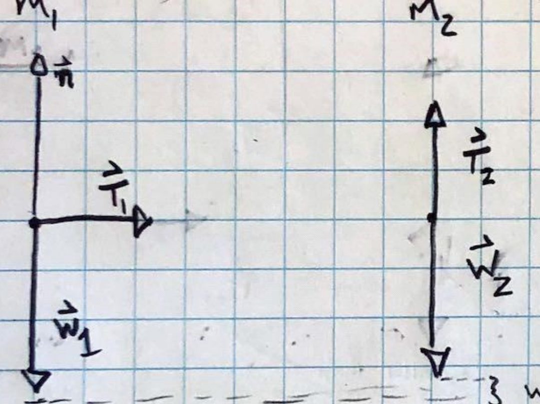
+ pulley

mass of string = 0

Mass of Pulley = 0







$w_1 > w_2 \Rightarrow m_1 > m_2$

therefore

$$F_{net,m_1} = \begin{cases} F_{y,m_1} = -w_1 + n = 0 \Rightarrow n = mg \; ; \; \therefore \; a_{m_1,y} = 0 \\ F_{x,m_1} = T_1 = m_1 a_{x1} \end{cases}$$

$T_1 = T_2$

$a_{x1} = -a_{y2}$

$$F_{net,m_2} = F_{y,m_2} = -w_2 + T_2 = -mg + T_2 = m_2 a_{y2}$$

Combine:

$$-m_2 g + (T_1 = m_1 a_{x1}) = m_2 a_{y2}$$

$$-m_2 g = m_2 a_{y2} - m_1 a_{x1}$$

$$-m_2 g = (m_2 + m_1) a_{y2}$$

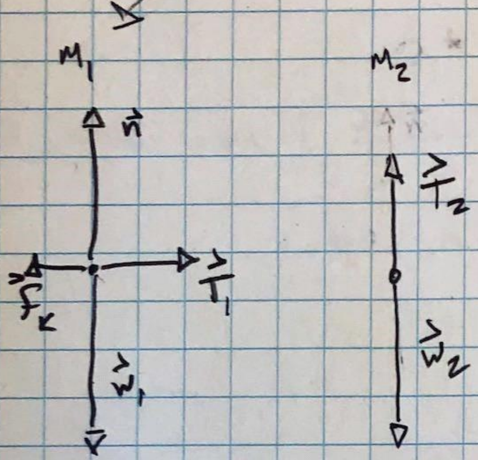
$$\therefore \boxed{a_{y2} = \frac{m_2}{(m_2 + m_1)}}$$

IF we include **Kinetic Friction** for block M_1:

$$\vec{f}_k = -\mu_k m_1 g$$

block M_1 accelerates to the right so, friction points to the left.

opposes motion!



$$F_{net_1} \begin{cases} F_{y_1} = 0 = m_1 a_{y_1} \\ F_{x_1} = \vec{T}_1 + \vec{f}_k = T_1 - \mu_k m_1 g = \left(m_1 a_{x_1} = -m_1 a_{y2} \right) \end{cases}$$

$$F_{net_2} \Rightarrow F_{y2} = \vec{T}_2 + \vec{w}_2 = T_1 - m_2 g = m_2 a_{y2}$$

Remove $\vec{T}$:

$$\left(F_{y2} - F_{x_1}\right) = (m_2 + m_1)\, a_{y2}$$

$$-m_2 g + \mu_k m_1 g = (m_2 + m_1)\, a_{y2}$$

$$\therefore \quad \boxed{a_{y2} = \frac{(-m_2 + \mu_k m_1)\, g}{(M_2 + M_1)}}$$

friction **reduces** a_{y2}!