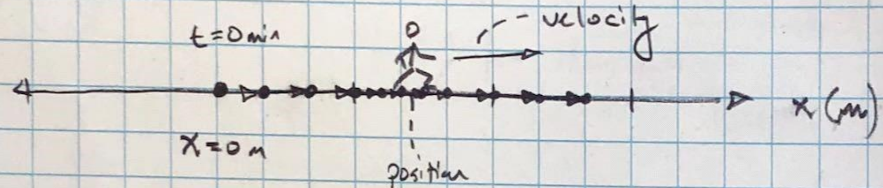
Ch 2 1D Motion

Kinematics = description of motion

→ Kinema (Greek) = movement

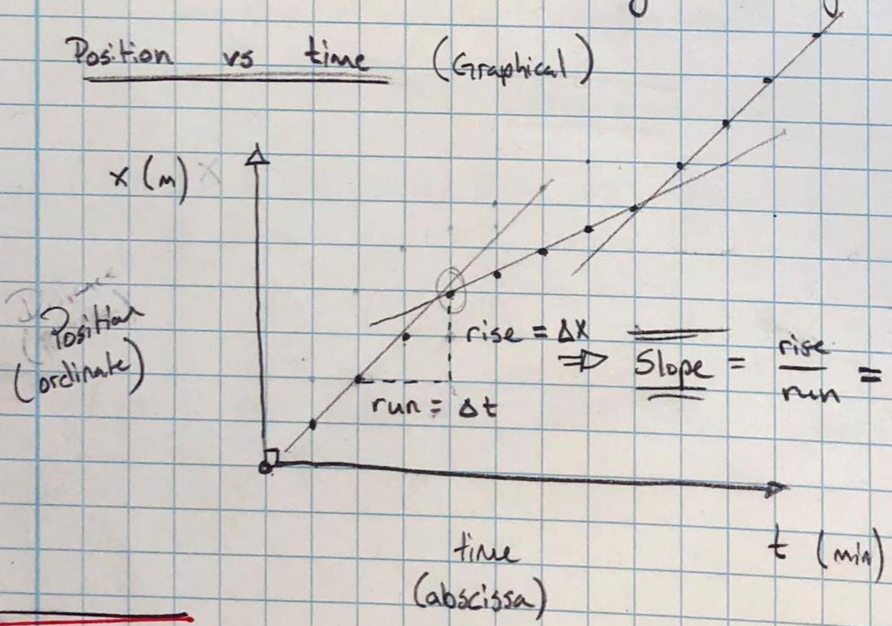
Kinematic variables = x (position), v (velocity)

↓ measured w.r.t. a coordinate system



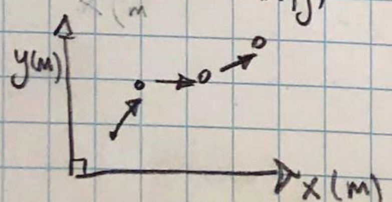
1D - motion = described by one physical coordinate

Position vs time (Graphical)



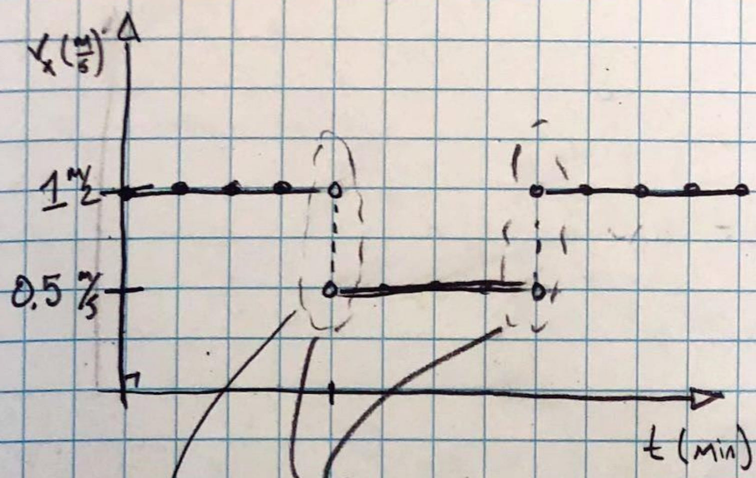
$\Rightarrow$ Slope $= \frac{\text{rise}}{\text{run}} =$ **Velocity**

2D - motion (x, y)

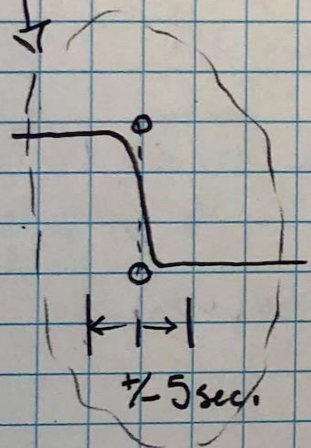


$$\left.\begin{aligned} v_x &= \frac{\Delta x}{\Delta t} \\ v_y &= \frac{\Delta y}{\Delta t} \end{aligned}\right\} |v| = \sqrt{v_x^2 + v_y^2}$$

Velocity vs time:



Velocity is discontinuous



recover position using definition of velocity:

$$V_x = \frac{\Delta x}{\Delta t} \; ; \quad \therefore \quad \Delta x = V_x \cdot \Delta t$$

if $V \neq$ constant, then add all increments

$$\left.\begin{aligned} \Delta x_1 &= V_{x_1} \Delta t_1 \\ \Delta x_2 &= V_{x_2} \Delta t_2 \\ &\vdots \\ \Delta x_N &= V_{x_N} \Delta t_N \end{aligned}\right\} \text{Sum them} \quad \Delta x = \sum_{n=1}^{N} V_{x_n} \Delta t_n = V_{x_1} \Delta t_1 + V_{x_2} \Delta t_2 \cdots V_{x_N} \Delta t_N$$

read: Sum from $n=1$ to N of $V_{x_n} \Delta t_n$

First equation of motion

recall:

$$\left\{ V_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_f} \right\} \cdot \Delta t$$

re-arrange: (multiply by Δt)

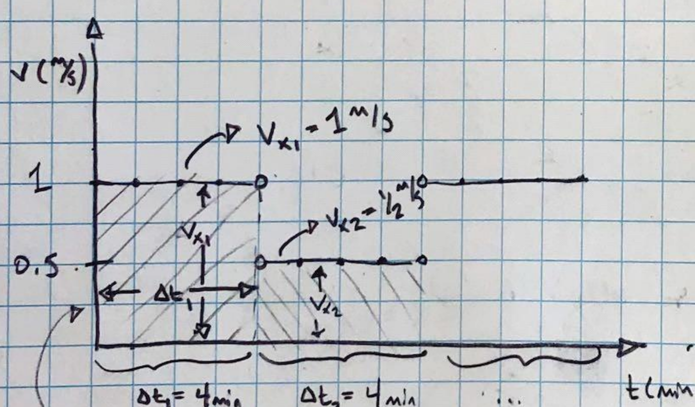
$$\left\{ V_x \Delta t = \Delta x = x_f - x_i \right\} + x_i$$

What is x_f? (Add x_i to both sides)

$$\underline{x_f = x_i + V_x \Delta t} \quad \Big\} \; x \propto \Delta t \; (x \text{ is Proportional to } \Delta t)$$

V_x = prop constant

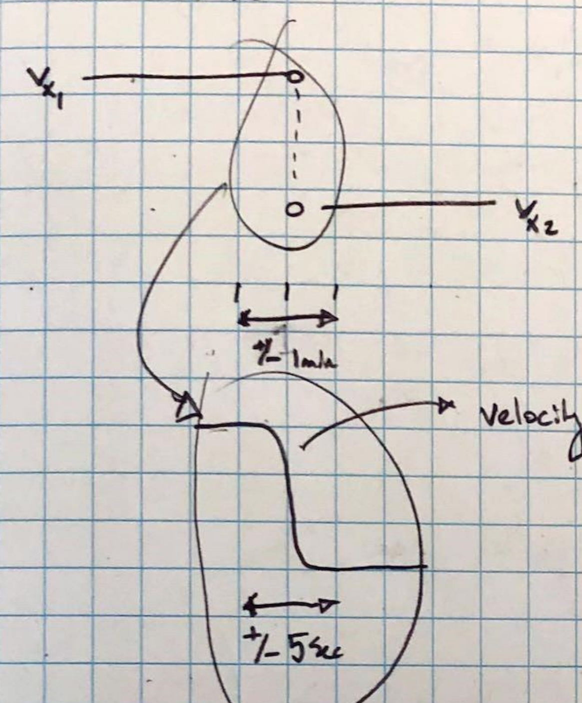
Displacement via "integration" (area under velocity curve)



$$\Delta x_1 = V_{x_1} \cdot \Delta t_1 = (\text{base}) \times (\text{height}) = \text{area under curve}$$

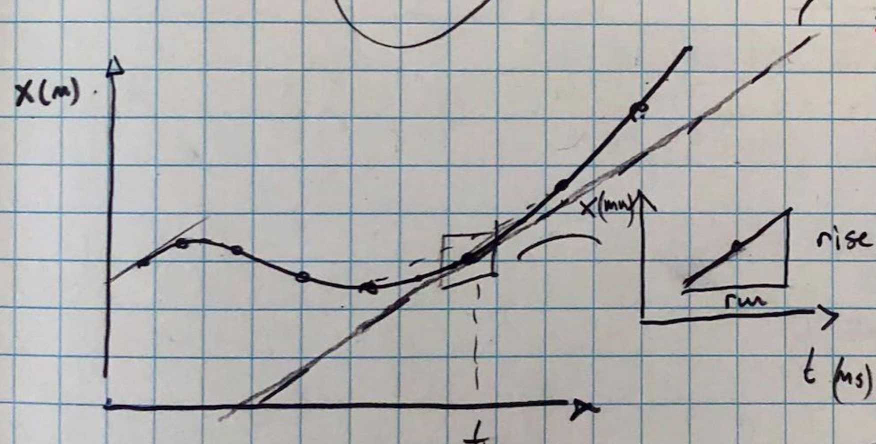
Instantaneous Velocity

recall that for

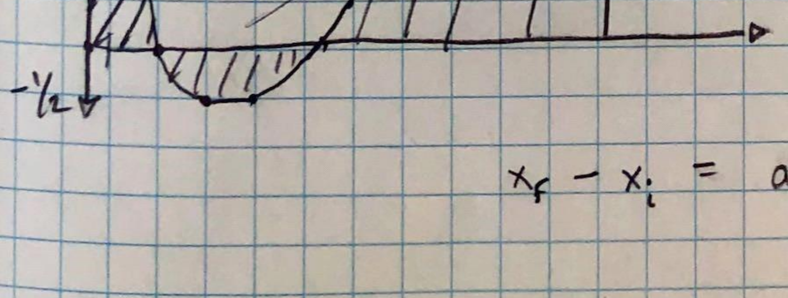


velocity is **continuous** (unique value at all times)

tangent = line w/ instantaneous slope $\left(\frac{\text{rise}}{\text{run}}\right)$

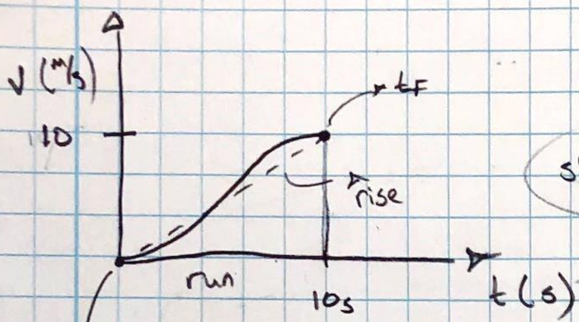


tangent = derivative of position @ t_0



$x_f - x_i$ = area under curve $v(t)$ from t_i to t_f

Acceleration (rate of change of velocity)



t_i = Stopped @ trafic light

Slope = $\frac{\text{rise}}{\text{run}}$

$$\Delta V = V_f - V_i$$

$$a \text{ (acceleration)} = \frac{\Delta V}{\Delta t}$$

Say $\Delta V = 10 \text{ m/s}$ & $\Delta t = 10 \text{ s}$

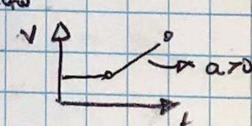
$$a = 1 \text{ m/s/s} = 1 \text{ m/s}^2$$

$$1g = 9.81 \text{ m/s}^2$$

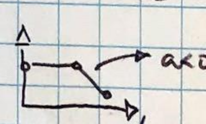
↳ "gravitational acceleration"

$\vec{a}$ is a vector, because ΔV is a vector

iF $\vec{v} > 0$ & $\vec{a} > 0$ = speeding up



iF $\vec{v} > 0$ & $\vec{a} < 0$ = Slowing down



Position from Constant Acceleration

if $\vec{a} = \text{const}$ (recall $\vec{a} = \frac{\Delta v}{\Delta t}$)

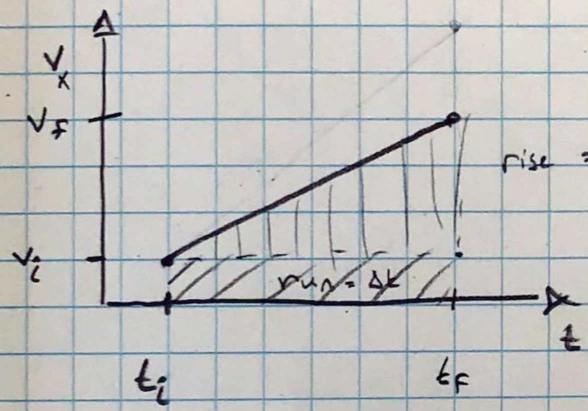
then,

$$\Delta v = a\,\Delta t$$

or

$$\boxed{v_{xf} = v_{xi} + a\,\Delta t = v_{xi} + a(t_f - t_i)}$$

a: slope of velocity curve



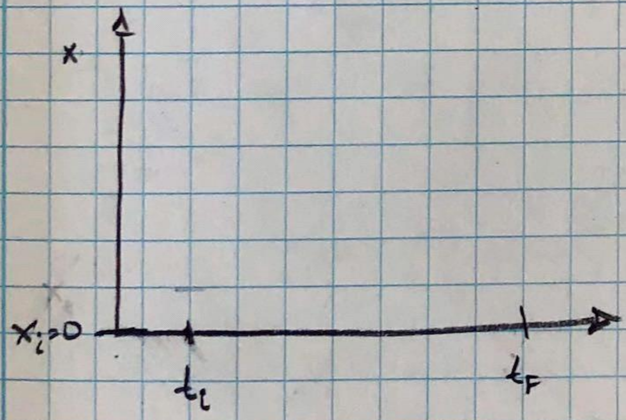
rise = $a \cdot \Delta t$

Δx = area under curve

$= v_i\,\Delta t$

$= \frac{1}{2}\Delta v\,\Delta t = \frac{1}{2} a\,\Delta t\,\Delta t$

$= \frac{1}{2} a (\Delta t)^2$



$$\boxed{x_f = \overset{0}{x_i} + v_i\,\Delta t + \frac{1}{2} a (\Delta t)^2}$$

Manipulation $\Longrightarrow$ $\Delta t = \frac{\Delta v}{a}$

$$\Delta x = v_i \cdot (v_f - v_i)/a + \frac{1}{2} a \left(\frac{(v_f - v_i)^2}{a^2}\right)$$

$$= \frac{v_i v_f}{a} - \frac{v_i^2}{a} + \frac{1}{2}\left(\frac{1}{a}\right)\left(v_f^2 - 2 v_f v_i + v_i^2\right)$$

$$= \frac{1}{2} v_f^2/a + \cancel{\frac{v_i v_f}{a}} - \cancel{\frac{v_f v_i}{a}} - \frac{1}{2} v_i^2/a \;; \qquad \boxed{v_f^2 = v_i^2 + 2 a_x \Delta x}$$

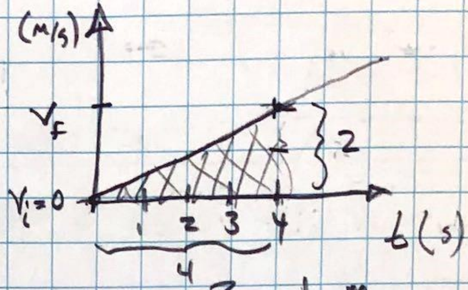
Constant acceleration from rest

$\vec{a} = a_i$

$v_i = 0$

$\Delta v = a_i \Delta t$

$v = a_i (t - t_i)$



$a_i = \frac{2}{4} = \frac{1}{2}\ m/s^2$

$v_f = a_i \Delta t$

$$\underbrace{\Delta x = \overset{\text{area under v}}{\triangle} = \frac{1}{2} a_i (\Delta t)^2}$$

quadratic $\quad x \sim t^2$

Proportionality

$x \propto y$ (x is proportional to y)

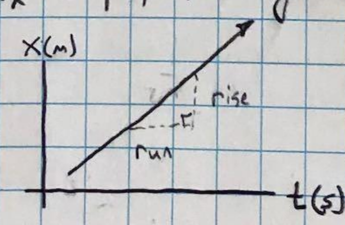
$x = C\, y$

Uniform Motion

$\Delta x = v_x \Delta t$

$\Delta x \propto \Delta t$

v_x is proportionality constant



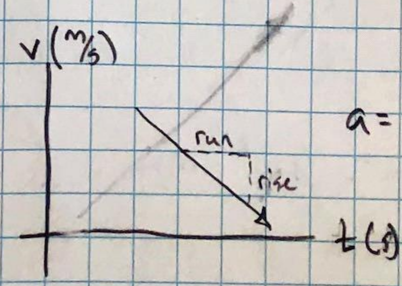
$v_x = \frac{\text{rise}}{\text{run}}$

Constant Acceleration

$\Delta v = a \Delta t$

$\Delta v \propto \Delta t$

a (acceleration) is proportionality constant



$a = \frac{\text{rise}}{\text{run}}$