CH 10: Work + Energy

Energy is neither created nor destroyed!

Can be converted from Potential ⟷ Kinetic → Internal (e.g. Heat)

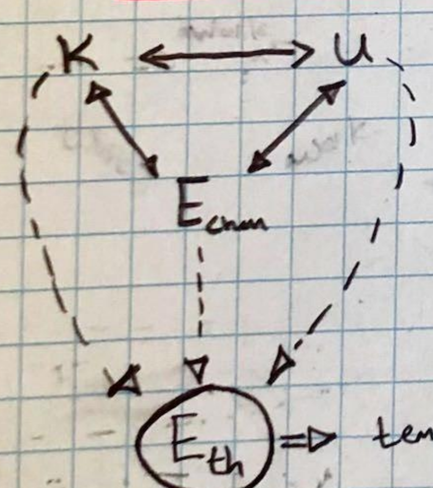
total Energy

$$\boxed{E = K + U_g + U_s + E_{th} + E_{chem}}$$

Kinetic (motion) Potential (gravity), (spring) Thermal Chemical (Bonds)

Work: transfer of mechanical energy; Kinetic ⟷ Potential

Heat: transfer of thermal energy between warmer / colder objects



$\Rightarrow$ temperature increases as ---▷ mechanical energy is dissipated.

work changes Energy

$$\boxed{W = \Delta E}$$

For an isolated system, $W = 0$

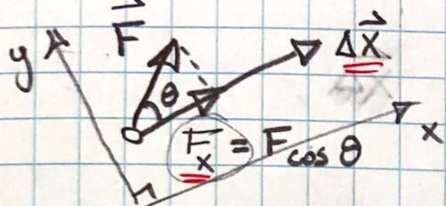
so, $\boxed{\Delta E = 0}$

$E_i = E_f$ ... but, energy can be transformed

<u>Work</u>: product of $\vec{\text{Force}}$ × $\overrightarrow{\text{displacement}}$

↳ parallel

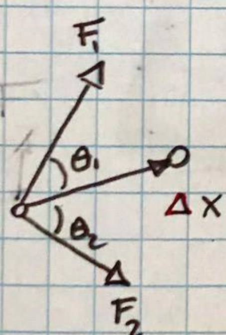
↳ force in the direction of displacement



$$\boxed{W_x = F_x \cdot \Delta X} \qquad \boxed{W_y = F_y \cdot 0 = 0}$$

$$W = \underbrace{F_{\parallel}}_{\text{parallel component of } F} \cdot \Delta X = F \Delta X \cos\theta$$

θ: angle between $\vec{F}$ & $\Delta\vec{X}$



$$\Rightarrow \boxed{\begin{aligned} W_{net} &= F_{\parallel,1}\,\Delta X + F_{\parallel,2}\,\Delta X + \dots \\ &= \Delta X \left\{ F_1 \cos\theta_1 + F_2 \cos\theta_2 + \dots \right\} \end{aligned}}$$

units: $Kg \frac{m}{s^2} \cdot m = N \cdot m = \boxed{J \text{ (Joules)}}$

Kinetic Energy: Energy of a body in motion!

$$\underbrace{W}_{\text{work}} = \underbrace{\Delta K}_{\text{change in kinetic energy}}$$

kinetic → motion

$$F \cdot \Delta x$$

recall: $(v_f)^2 - (v_i)^2 = 2 a \Delta x$

Kinematics: constant acceleration (a)

Newton's 2nd Law:

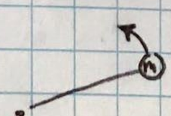
$$\frac{F}{m} = a$$

$$\therefore \quad \boxed{F \cdot \Delta x = \frac{1}{2} m \left\{ v_f^2 - v_i^2 \right\} = \Delta K = K_f - K_i}$$

$$\boxed{K = \frac{1}{2} m v^2}$$

units: $\text{kg} \, \frac{m^2}{s^2} = N \cdot m$ → work: $F \cdot \Delta x$ ✓

Rotational Kinetic Energy: $v = \omega r$



$$K_{rot} = \frac{1}{2} m (\omega r)^2$$

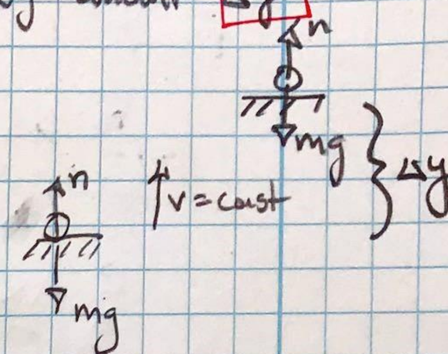
$$= \frac{1}{2} \underbrace{(mr^2)}_{I} \omega$$

$$\boxed{K_{rot} = \frac{1}{2} \left(\underbrace{m_1 r_1^2 + m_2 r_2^2 + \dots}_{I} \right) \omega^2 = \frac{1}{2} I \omega^2}$$

Potential Energy:

Gravitational Potential U_g:

consider mass (M) raised @ constant speed by amount $\boxed{\Delta y}$



$$F_{net} = n - mg = 0$$

$$n = mg$$

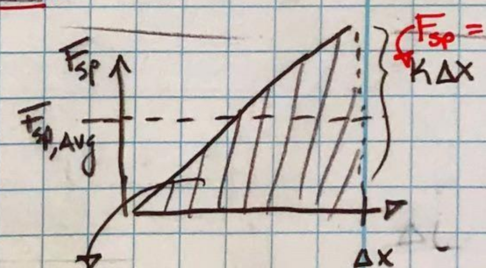
assume your hand is an external agent, but the earth is part of the system

$$\boxed{F_{net,\ external} = n = mg}$$

$$\underbrace{F_{n,e} \cdot \Delta y} = U_{g,F} - U_{g,i}$$

$$\boxed{U_{g,F} = U_{g,i} + mg\,\Delta y}$$

Spring Potential: U_{sp}



$$W = U_{sp,F} - U_{sp,i} = F_{sp,Avg} \cdot \Delta X$$

$$A = \frac{1}{2} K \Delta X^2 = F_{sp,avg}\,\Delta X$$

$$\boxed{U_{sp,F} = U_{sp,i} + \frac{1}{2} K \Delta X^2}$$

Friction: Conversion of mechanical to heat energy

$$W = \Delta E_{th}$$

$$F \cdot \Delta x =$$

$$f_k \Delta x =$$

$$\boxed{\mu_k n \Delta x = \Delta E_{th}}$$

Conservation: $\Rightarrow$ $\boxed{W = E_f - E_i}$

$$E_f = E_i + W$$

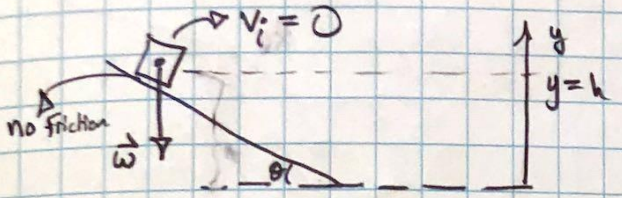
open system

$$\boxed{K_f + U_f + \Delta E_{th} = K_i + U_i + W}$$

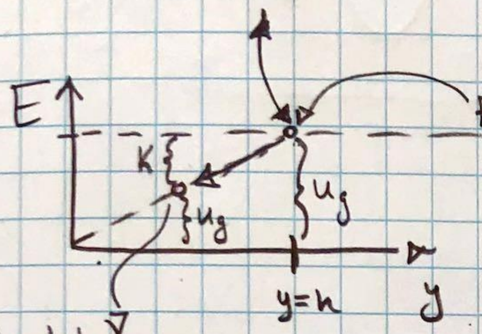
$W = 0$ for isolated system

$$\boxed{K_f + U_f + \Delta E_{th} = K_i + U_i}$$

Energy Diagram:

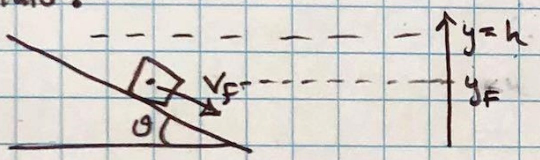


$E_i = U_{g,i} = mgh$



$E_i = E_f = mgh$

Some time later:

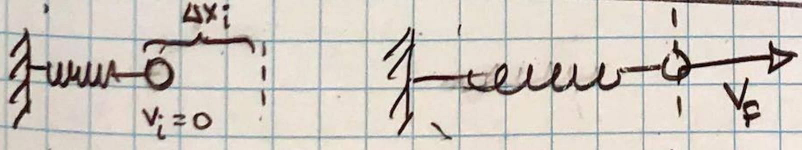


$$E_f = \underbrace{U_{g,f}}_{mg\,y_f} + \underbrace{K_f}_{\frac{1}{2} m v_f^2}$$

$$E_f = E_i$$

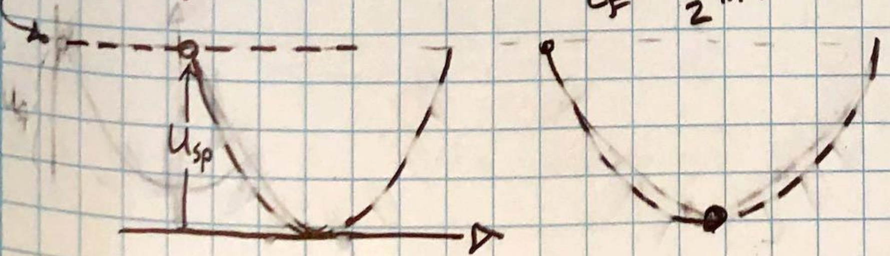
$$\frac{1}{2} \cancel{m} v_f^2 = \cancel{m} g (h - y_f)$$

$$v_f = \sqrt{2g(h - y_f)}$$



$E_i = \frac{1}{2} k \Delta x_i^2$

$E_f = \frac{1}{2} m v^2$

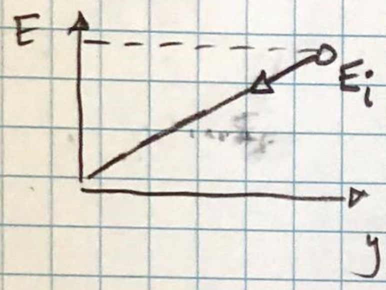


$$E_f = E_i$$

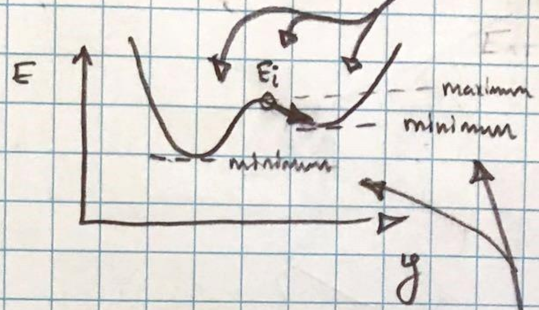
$$\frac{1}{2} m v_f^2 = \frac{1}{2} k \Delta x_i^2 \Rightarrow \boxed{v_f = \sqrt{\frac{k}{m}}\, \Delta x_i}$$

Equilibrium : System wants to move toward the nearest minimum in the Energy diagram

Extrema of Energy diagram



or

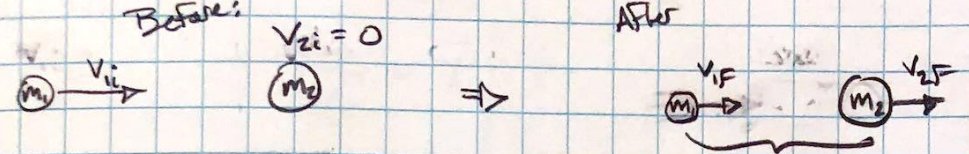


An unstable Equilibrium is a local maxima:

a small perturbation will knock it out of equilibrium

Stable Equilibrium is a local minimum...

Elastic Collision:



$$P_{tot,i} = m_1 v_{1i}$$

$$\boxed{\Delta P_{tot} = 0}$$

$$P_{tot,F} = m_1 v_{1F} + m_2 v_{2F}$$

$$\underbrace{v_{1F} \neq v_{2F}}_{\text{Elastic}} \quad (\text{inelastic } v_{1F} = v_{2F})$$

$$\boxed{m_1 v_{1F} = m_1 v_{1i} - m_2 v_{2F}} \quad (1)$$

$$\Delta K = K_f - K_i$$

$$\underbrace{\tfrac{1}{2} m_1 v_{1F}^2 + \tfrac{1}{2} m_2 v_{2F}^2}_{K_f} - m_1 v_{1i}^2$$

Energy is conserved in a perfectly Elastic Collision: $\Delta K = 0$

$$\boxed{\tfrac{1}{2} m_1 v_{1F}^2 = \tfrac{1}{2} m_1 v_{1i}^2 - \tfrac{1}{2} m_2 v_{F2}^2} \quad (2)$$

to get v_{2F}, sub v_F from (1) into (2) + solve for v_{2F}:

$$\boxed{v_{2F} = \frac{2m_1}{m_1 + m_2} v_{1i}}$$

to get v_{1F}, sub into (1) + solve for v_{1F}

$$\boxed{v_{1F} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}}$$

Perfectly Elastic Collision

if $m_2 > m_1$ the direction of v_1 changes

if $m_2 = m_1$, $v_{1F} = 0$!

Power: the rate of working = rate of change in energy

$$\boxed{P = \frac{W}{\Delta t}}$$

$$\boxed{P = \frac{\Delta E}{\Delta t}}$$

Units: Watt (W) $= \frac{J}{s} = kg\left(\frac{m}{s}\right)^2 \cdot \frac{1}{s}$

$$W = F_{\parallel} \cdot \Delta x$$

$$P = \frac{F_{\parallel}\,\Delta x}{\Delta t} = F_{\parallel}\frac{\Delta x}{\Delta t} = F_{\parallel} v$$