Homework #3, Problems: 17, 18, 21, 22, 29 in chapter 2 and 2, 4 in chapter 3

2-17 \( \Delta m = m_{Ra} - m_{Rn} - m_{He} \) (an atomic unit of mass, the u, is one-twelfth the mass of the \(^{12}\)C atom or 1.660 54 \( \times \) 10\(^{-27} \) kg)

\[
\Delta m = (226.025 4 - 22.017 5 - 4.002 6) \text{ u} = 0.005 3 \text{ u}
\]

\[
E = (\Delta m)(931 \text{ MeV/u}) = (0.005 3 \text{ u})(931 \text{ MeV/u}) = 4.9 \text{ MeV}
\]

2-18 (a) The mass difference of the two nuclei is

\[
\Delta m = 54.927 9 \text{ u} - 54.924 4 \text{ u} = 0.003 5 \text{ u}
\]

\[
\Delta E = (931 \text{ MeV/u})(0.003 5 \text{ u}) = 3.26 \text{ MeV}.
\]

(b) The rest energy for an electron is 0.511 MeV. Therefore,

\[
K = 3.26 \text{ MeV} - 0.511 \text{ MeV} = 2.75 \text{ MeV}.
\]

2-21

\[
\begin{align*}
e(-) & \quad e(+) \\
K, p(e-) & \quad \text{positron at rest}
\end{align*}
\]

\[
E, p
\]

\[
\gamma, \theta
\]

Conservation of mass-energy requires \( K + 2mc^2 = 2E \) where \( K \) is the electron’s kinetic energy, \( m \) is the electron’s mass, and \( E \) is the gamma ray’s energy.

\[
E = \frac{K}{2} + mc^2 = (0.500 + 0.511) \text{ MeV} = 1.011 \text{ MeV}.
\]

Conservation of momentum requires that \( p_e = 2p \cos \theta \) where \( p_e \) is the initial momentum of the electron and \( p \) is the gamma ray’s momentum, \( \frac{E}{c} = 1.011 \text{ MeV}/c \). Using \( E^2 = p^2c^2 + (mc^2)^2 \)

where \( E_e \) is the electron’s total energy, \( E_e = K + mc^2 \), yields

\[
p_e = \frac{1}{c} \sqrt{K^2 + 2Kmc^2} = \frac{\sqrt{(1.00)^2 + 2(1.00)(0.511) \text{ MeV}}}{c} = 1.422 \text{ MeV}/c.
\]

Finally, \( \cos \theta = \frac{p_e}{2p} = 0.703 \); \( \theta = 45.3^\circ \).

2-22 (a) Using the results of Problem 2-6 and substituting numerical values
\[ p \text{ (in MeV/c)} = 300BR = (300)(2.00 \text{ T})(0.343 \text{ m}) = 206 \text{ MeV/c}. \]

Since the momentum of the \( K^0 \) is zero before the decay, conservation of momentum requires the pion momenta to be equal in magnitude and opposite in direction. The pion’s speed \( u \) may be found by noting that
\[
\frac{p}{E} = \frac{mu}{mc^2} = \sqrt{1 - \frac{u^2}{c^2}} \quad \text{or} \quad u = \frac{pc}{E} \quad \text{where} \quad p \text{ is the pion momentum and } E \text{ is the pion’s total energy.} \]

Thus for either pion,
\[
\frac{u}{c} = \frac{pc}{E} = \frac{206 \text{ MeV}}{\sqrt{(206 \text{ MeV})^2 + (104 \text{ MeV})^2}} = 0.827. \]

\( \text{(b) Conservation of mass-energy requires that } E_{K^0} = 2E \text{ where } E_{K^0} \text{ is the total energy of a pion. As the } K^0 \text{ pion decays at rest,}
\]
\[
E_{K^0} = m_{K^0}c^2 = 2\sqrt{p^2c^2 + \left(mc^2\right)^2} = 2\sqrt{(206)^2 + (140)^2} \text{ MeV} = 498 \text{ MeV},
\]

or \( m_{K^0} = 498 \text{ MeV/c}^2 \).

2-29 The energy of the first fragment is given by \( E_1^2 = p_x^2c^2 + \left(m_1c^2\right)^2 = (1.75 \text{ MeV})^2 + (1.00 \text{ MeV})^2; \)
\( E_1 = 2.02 \text{ MeV}. \) For the second, \( E_2^2 = (2.00 \text{ MeV})^2 + (1.50 \text{ MeV})^2; \)
\( E_2 = 2.50 \text{ MeV}. \)

(a) Energy is conserved, so the unstable object had \( E = 4.52 \text{ MeV} \). Each component of momentum is conserved, so the original object moved with
\[
p^2 = p_x^2 + p_y^2 = \left(\frac{1.75 \text{ MeV}}{c}\right)^2 + \left(\frac{2.00 \text{ MeV}}{c}\right)^2.
\]

Then for \( (4.52 \text{ MeV})^2 = (1.75 \text{ MeV})^2 + (2.00 \text{ MeV})^2 + (mc^2)^2 \); \( m = 3.65 \text{ MeV/c}^2 \).

(b) Now \( E = \gamma mc^2 \) gives \( 4.52 \text{ MeV} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ 3.65 MeV; } 1 - \frac{v^2}{c^2} = 0.654 \text{ and } v = 0.589c. \)

3-2 Assume that your skin can be considered a blackbody. One can then use Wien’s displacement law,
\[ \lambda_{\text{max}}T = 0.289 \times 10^{-2} \text{ m} \cdot \text{K} \text{ with } T = 35^\circ \text{C} = 308 \text{ K} \text{ to find}
\]
\[
\lambda_{\text{max}} = \frac{0.289 \times 10^{-2} \text{ m} \cdot \text{K}}{308 \text{ K}} = 9.41 \times 10^{-6} \text{ m} = 9.410 \text{ nm}.
\]
3-4 (a) From Stefan’s law, one has \( \frac{P}{A} = \sigma T^4 \). Therefore,

\[
\frac{P}{A} = (5.7 \times 10^{-8} \text{ W/m}^2 \text{K}^4)(3000 \text{ K})^4 = 4.62 \times 10^6 \text{ W/m}^2 .
\]

(b) \( A = \frac{P}{4.62 \times 10^6 \text{ W/m}^2} = \frac{75 \text{ W}}{4.62 \times 10^6 \text{ W/m}^2} = 16.2 \text{ mm}^2 . \)