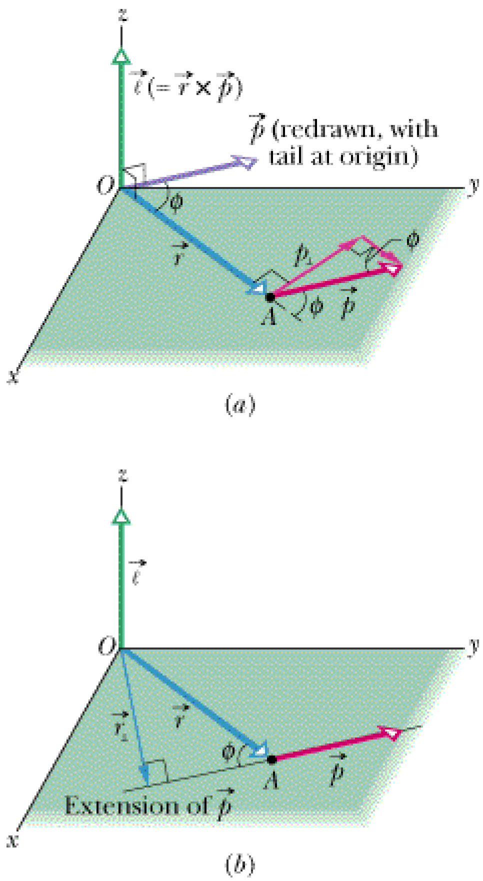
Chpter 12. Complex Rotations

1. Rotational Momentum
2. Rotational Form of Newton's Second Law
3. The Rotational Momentum of a System of Particles
4. The Rotational Momentum of a Rigid Body Rotating About a Fixed Axis
5. Conservation of Rotational Momentum

Rotational Momentum



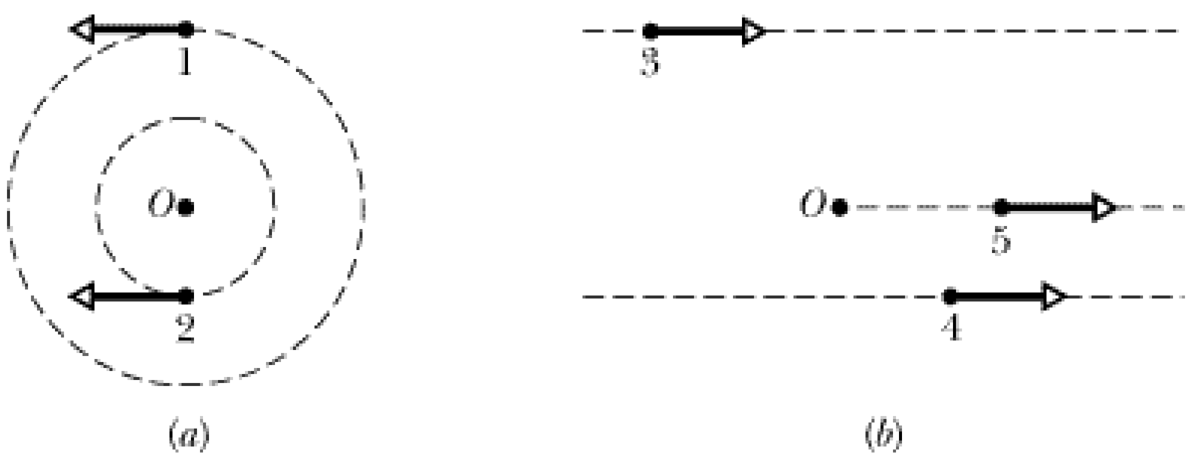
A particle of mass m with translational momentum $\vec{p} = m\vec{v}$ as it passes through point A in the xy plane. The **rotational momentum** $\vec{\ell}$ of this particle with respect to the origin O is a vector quantity defined as

$$\vec{\ell} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

where $\vec{r}$ is the position vector of the particle with respect to O.

Note: the particle does *not* have to rotate around O.

Excise: In the diagrams below there is an axis of rotation perpendicular to the page that intersects the page at point *O*. Figure (*a*) shows particles 1 and 2 moving around point *O* in opposite rotational directions, in circles with radii 2 m and 4 m. Figure (*b*) shows particles 3 and 4 traveling in the same direction, along straight lines at perpendicular distances of 2 m and 4 m from point *O*. Particle 5 moves directly away from *O*. All five particles have the same mass and the same constant speed. (a) Rank the particles according to the magnitudes of their rotational momentum about point *O*, greatest first. (b) Which particles have rotational momentum about point *O* that is directed into the page?



Rotational Form of Newton's Second Law

The (vector) sum of all the torques acting on a particle is equal to the time rate of change of the rotational momentum of that particle.

$$\vec{\tau}^{\,net} = \frac{d\vec{\ell}}{dt}$$

The Rotational Momentum of a System of Particles

The total rotational momentum $\vec{L}$ of a system of particles to be the vector sum of the rotational momenta $\vec{\ell}$ of the individual particles

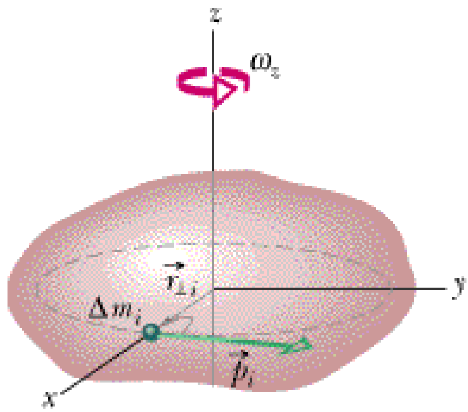
$$\vec{L} = \vec{\ell}_A + \vec{\ell}_B + \vec{\ell}_C + \cdots + \vec{\ell}_n = \sum_{i=A}^{n} \vec{\ell}_i,$$

Newton's Law: The net (external) torque $\vec{\tau}^{net}$ acting on a system of particles is equal to the time rate of change of the system's total rotational momentum $\vec{L}$.

$$\vec{\tau}^{net} = \frac{d\vec{L}}{dt} \quad \text{(system of particles)}, \tag{12-32}$$

where $\vec{\tau}^{net}$ is the net torque acting on the system.

The Rotational Momentum of a Rigid Body Rotating About a Fixed Axis



DEFINITION OF ANGULAR MOMENTUM

The angular momentum L of a body rotating about a fixed axis is the product of the body's moment of inertia I and its angular velocity ω with respect to that axis: $\vec{L} = I\vec{\omega}$

Unit of Angular Momentum: kg·m^2/s	(9.10)

PRINCIPLE OF CONSERVATION OF Rotational MOMENTUM

The total angular momentum of a system remains constant (is conserved) if the net external torque acting on the system is zero.

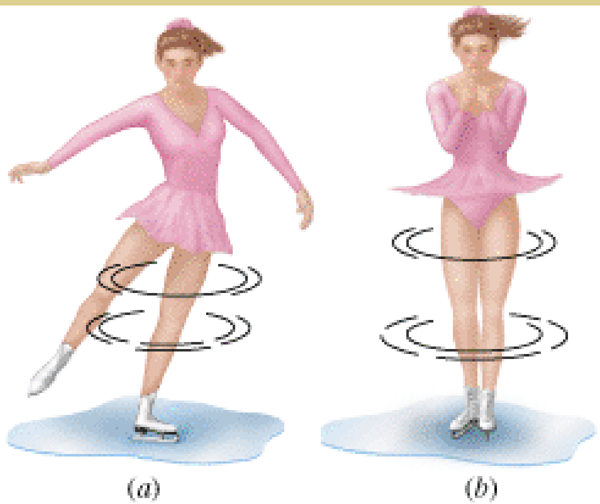


TABLE 12-2

More Corresponding Relations for Translational and Rotational Motion*

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau} = \vec{r} \times \vec{F}$
Translational momentum	$\vec{p}^{\,sys}$	Rotational momentum	$\vec{\ell} = \vec{r} \times \vec{p}$
Translational momentum[a]	$\vec{p}^{\,sys} = \Sigma \vec{p}_i$	Rotational momentum[a]	$\vec{L} = \Sigma \vec{\ell}_i$
Translational momentum[a]	$\vec{p}^{\,sys} = M\vec{v}_{com}$	Rotational momentum[b]	$\vec{L} = I\vec{\omega}$
Newton's Second Law[a]	$\Sigma \vec{F}^{\,ext} = \dfrac{d\vec{p}^{\,sys}}{dt}$	Newton's Second Law[a]	$\Sigma \vec{\tau}^{\,ext} = \dfrac{d\vec{L}}{dt}$
Conservation law[c]	$\vec{p}^{\,sys} =$ a constant	Conservation law[c]	$\vec{L} =$ a constant

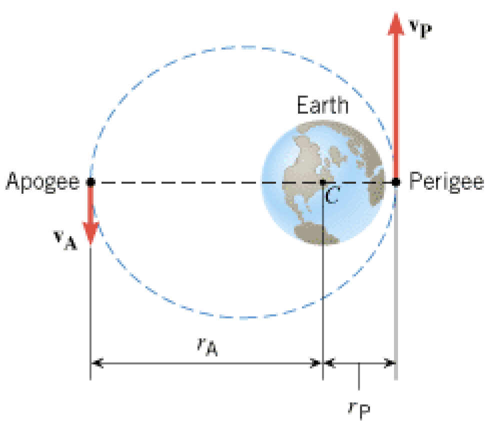
[a] For systems of particles, including rigid bodies.

[b] For a rigid body about a fixed axis, with L being the component along that axis.

[c] For a closed, isolated system ($\vec{F}^{\,net} = 0$, $\vec{\tau}^{\,net} = 0$).

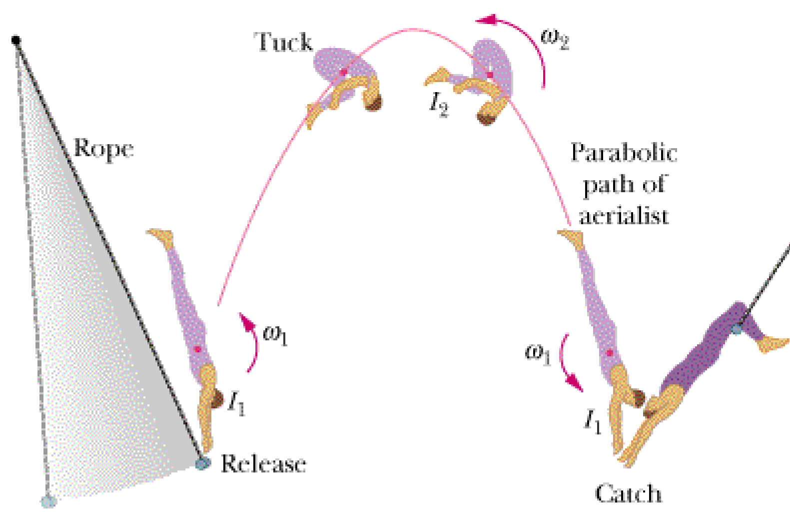
Example 1 A Satellite in an Elliptical Orbit

An artificial satellite is placed into an elliptical orbit about the earth, as in Figure 9.27. Telemetry data indicate that its point of closest approach (called the *perigee*) is $r_P = 8.37 \times 10^6$ m from the center of the earth, and its point of greatest distance (called the *apogee*) is $r_A = 25.1 \times 10^6$ m from the center of the earth. The speed of the satellite at the perigee is $v_P =$ 8450 m/s. Find its speed v_A at the apogee.



TOUCHSTONE EXAMPLE 2: Quadruple Somersault

During a jump to his partner, an aerialist is to make a quadruple somersault lasting a time $t = 1.87$ s. For the first and last quarter revolution, he is in the extended orientation shown in Fig. 12-20, with rotational inertia $I_1 = 19.9 \text{ kg} \cdot \text{m}^2$ around his center of mass (the dot). During the rest of the flight he is in a tight tuck, with rotational inertia $I_2 = 3.93 \text{ kg} \cdot \text{m}^2$. What must be his rotational speed ω_2 around his center of mass during the tuck?



Conceptual Questions

1

A woman is sitting on the spinning seat of a piano stool with her arms folded. What happens to her (a) angular velocity and (b) angular momentum when she extends her arms outward? Justify your answers.

2

A person is hanging motionless from a vertical rope over a swimming pool. She lets go of the rope and drops straight down. After letting go, is it possible for her to curl into a ball and start spinning? Justify your answer.