

PHYSICS 102 LAB 11: ATOMIC SPECTRA
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THEORETICAL DISCUSSION

All photons have a definite energy E_γ , which can be written in terms of the photon wavelength λ and frequency ν as

$$E_\gamma = h\nu = \frac{hc}{\lambda}$$

where $c \approx 3 \times 10^8$ m/s is the speed of light and h is a constant called *Planck's constant*. The numerical value of Planck's constant is

$$h = (6.62606876 \pm 0.00000052) \times 10^{-34} \text{ J} \cdot \text{s}$$

in SI units, or

$$h = (4.13566727 \pm 0.00000016) \times 10^{-15} \text{ eV} \cdot \text{s}$$

in atomic units. In either case, h has the dimensions of an energy times a time and it is, along with the speed of light c and the fine structure constant α , one of the most fundamental and important physical constants known to modern physics. It's not obvious, but h also has the same dimensions as angular momentum[1]. It was Niels Bohr's hypothesis that the angular momentum of an electron in orbit around a nucleus is quantized in units of h that led him to propose the so-called *Bohr model* of the hydrogen atom. This model explained the discrete light spectrum emitted by an electrically excited hydrogen plasma perfectly, and, although it was later shown to be an incomplete description of atomic spectra in general, it provided the first theoretical description of an otherwise unfathomable phenomenon in terms of the newly developed quantum theory. Our lab today entails observing and making measurements of the hydrogen spectrum in order to verify the essential elements of Bohr's atomic model.

Quantizing the electron orbit: Although a complete description of the Bohr model is far beyond the scope of this course, the fundamentals of the theory are surprisingly easy to grasp. If you assume that the electron follows a circular orbit around the nucleus[2] the angular momentum L of the electron is equal to

$$L = mvr$$

where m is the electron mass, v is the tangential velocity of the electron in its orbit and r is the orbital radius. The magnitude of the electric force acting on the electron due to the proton is equal to

$$F = \frac{e^2}{4\pi\epsilon_0 r^2}$$

where $e = 1.602 \times 10^{-19}$ C is the magnitude of the charge of both the electron and the proton. You are probably more familiar with seeing the force written in the form

$$F = \frac{ke^2}{r^2}$$

These two forms are equivalent, because in fact,

$$k = \frac{1}{4\pi\epsilon_0}$$

There are two reasons why we are not using the constant k in this lab write-up.

1. The constant k is typically not used by working scientists. It is really only used in algebra-based introductory physics classes. The rest of the world uses $\frac{1}{4\pi\epsilon_0}$.
2. The Physics 202 students have to use the form $\frac{1}{4\pi\epsilon_0}$, both because many of them in the future will have to work with chemists and physicists who also use that form, and because their textbooks use it. During the summer, we have a mixed lab group, including both Physics 102 and Physics 202 students. Unless we wish half the class to be crunching one set of numbers and the other half crunching a different set (which we don't), we have to choose one or the other way of writing this factor. Since the form $F = \frac{e^2}{4\pi\epsilon_0 r^2}$ is the standard one, we are inclined to prefer it. Sorry if this causes you any confusion. Your lab instructor will be happy to help out if this is the case. Historically, there have not been any problems with this issue.

For an object in uniform circular motion, the central force is related to the orbital velocity and the orbital radius according to

$$F = ma = \frac{mv^2}{r}$$

Combining the previous two equations gives

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r}$$

which you can solve for v^2 to get

$$v^2 = \frac{e^2}{4\pi\epsilon_0 mr}$$

Bohr's hypothesis that the angular momentum of the electron's orbital motion is quantized means that only certain values of L are allowed. In particular, the angular momentum can only have the values

$$L = mvr = \frac{nh}{2\pi} \quad n = 1, 2, 3, \dots$$

You can write the quantization condition in terms of v as

$$v = \frac{nh}{2\pi mr}$$

and then substitute this into the expression for v^2 above to get a quantization condition for the radius r :

$$v^2 = \frac{e^2}{4\pi\epsilon_0 mr} = \frac{n^2 h^2}{4\pi^2 m^2 r^2}$$

so that

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

where the label n on the radius tells you which quantum orbit the electron is in. The lowest energy orbital is the first, with $n = 1$; higher energy (more loosely bound) orbital radii can be calculated by inserting $n = 2, 3, 4, \dots$

Quantizing the electron energy levels: The kinetic energy K of the electron in its orbit is simply

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\epsilon_0 r}$$

where we have substituted the expression for v^2 that we derived from the expression for the relation between the central force and the orbital velocity. The potential energy U can be found from Coulomb's law. It is

$$U = -\frac{e^2}{4\pi\epsilon_0 r}$$

The total energy E is just the sum of the kinetic and potential energies.

$$E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r}$$

By substituting the expression for the quantized radii in the above expression, we get the energy levels of the hydrogen atom:

$$E_n = -\frac{me^4}{8h^2\epsilon_0^2 n^2} \quad n = 1, 2, 3, \dots$$

The hydrogen line spectrum: When an electron changes from one energy level to another, the energy of the atom must change as well. It requires energy to promote an electron from one energy level to a higher one. This energy can be supplied by a photon whose energy E_γ is given in terms of its frequency ν or wavelength λ as

$$E_\gamma = h\nu = \frac{hc}{\lambda}$$

Since the energy levels are quantized, only certain photon wavelengths can be absorbed. If energy is put into the atomic gas, the electrons will be promoted to higher energy levels and will then fall back down into the lowest energy state (ground state) in a cascade of transitions. Each time the energy level of the electron changes, a photon will be emitted and the energy (wavelength) of the photon will be characteristic of the energy difference between the initial and final energy levels of the atom in the transition. The energy of the emitted photon is just the difference between the energy levels of the initial (n_i) and final (n_f) states.

$$E_\gamma = \frac{hc}{\lambda} = E_{n_i} - E_{n_f} = -\frac{me^4}{8h^2\epsilon_0^2} \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

The set of spectral lines for a given final state n_f are generally close together. In the hydrogen atom they are given special names. The states for which $n_f = 1$ are called the *Lyman series*. These transitions are in the ultraviolet. The states for which $n_f = 2$ are called the *Balmer series* and many of these spectral lines are in the visible. We will be measuring the wavelengths of the Balmer series lines in today's lab.

PROCEDURE AND ANALYSIS

The "colors" associated with some of the Balmer series lines are shown in figure 1.

n_i	“color”
3	red
4	blue-green
5	blue
6	violet



FIG. 1: The Balmer series lines of the hydrogen spectrum

1. Measure the wavelengths for the $n_i = 3, 4, 5$, and 6 lines in the Balmer series of hydrogen using the diffraction grating and the spectrometer. Recall that the relation between the wavelength and the observation angle θ for the 1st order images ($m = 1$) is given by the grating equation:

$$d \sin \theta = \lambda$$

The measurement will be very similar to last week's lab except in this case you will use the angle table on the spectrometer rather than trigonometry to determine the observation angles for the various spectral lines. Bear in mind that the observation angle is always measured with respect to the normal to the diffraction grating. A diagram of the spectrometer table is shown in figure 2.

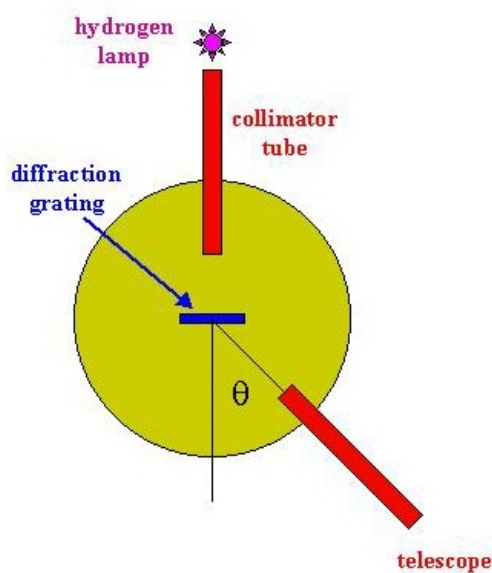


FIG. 2: The spectrometer table

2. Calculate the photon energies for each of your measured lines using the relation

$$E_\gamma = \frac{hc}{\lambda} \quad (1)$$

and report it in electron volts (eV).

3. Calculate the corresponding transition energies for the Balmer series lines from the equation

$$E_{n_i} - E_{n_f} = -\frac{me^4}{8h^2\epsilon_0^2} \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \quad (2)$$

where n_f has been set equal to 2. Report your transition energies in electron volts (eV).

4. Numerically compare your photon energies to your transition energies by calculating their fractional discrepancy.

USEFUL CONSTANTS AND PARAMETERS

The grating spacing is $d = \frac{1}{600}$ mm.

electron mass m	9.11×10^{-31} kg
electron charge e	1.602×10^{-19} C
electric permittivity constant ϵ_0	8.85×10^{-12} C ² /N · m ²
speed of light c	3.0×10^8 m/s

The value of Planck's constant was given at the beginning of the lab. The conversion factor between Joules and electron-volts is

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J} \quad (3)$$

REALLY VALUABLE HINTS!!

1. If you simply use the value of Planck's constant h as expressed in eV · s in equation 2, you are going to get the wrong answer. A clever and patient individual can use dimensional analysis to figure out why this is so. (Are you a clever and patient individual? It's a good thing to be if you want to be a scientist!) Instead, you are going to have to calculate the answer first in Joules, and then use the conversion factor from equation 3 to get your answer in eV. On the other hand, you can directly calculate the photon energies from equation 1 using the value of Planck's constant as expressed in eV · s. As before, dimensional analysis reveals why this works in this case.
2. For the different lines in the balmer series, only the factor $\left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$ in equation 2 changes. The factor $\frac{me^4}{8h^2\epsilon_0^2}$ is the same for each line. It makes sense to calculate this factor once at the beginning, and then multiply it by the other factors that you calculate for each separate line.
3. You will almost certainly discover that if you simply plug the numbers from the table of parameters into equation 2, your calculator will crash. This is because the numbers in the intermediate steps of the calculation are too big for most calculators to handle. The way around this problem is to separately calculate the mantissa and exponent for the numbers. (You may recall from high school that a number in scientific notation is written in the form $n \times 10^x$. In this equation, n is the mantissa and x is the exponent). So, supposing that you had to calculate the quantity

$$y = \frac{(n_1 \times 10^{x_1})(n_2 \times 10^{x_2})}{(n_3 \times 10^{x_3})(n_4 \times 10^{x_4})}$$

then one way to do it would be to calculate

$$y = \frac{(n_1)(n_2)}{(n_3)(n_4)} \times 10^{(x_1+x_2-x_3-x_4)}$$

Unless your calculator is REALLY expensive, this is how you're going to have to do it for this calculation.

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- [1] Angular momentum is linear momentum times a length, which works out to have the dimensions of mass \times (length)² / time. This is the same as energy \times time.
- [2] This assumption is not, strictly speaking, correct. However, the quantities we are interested in—the angular momentum, average velocity and electrostatic potential energy—can all be correctly calculated for the hydrogen atom as if this assumption were true. This is because, even though quantum mechanics tells us that the actual trajectory of any given electron in its orbit cannot be predicted in advance, or even observed without modifying it, the average orbit of an electron does follow the classic circular trajectory and we can calculate certain quantities as if the hydrogen atom were a classical system, subject to certain quantum mechanical constraints. A system for which quantum mechanical constraints can be applied *ex post facto* to the classical dynamical variables of that system is called *semi-classical*. The hydrogen atom is amenable to a semi-classical analysis, but atoms in general are not, and the derivation presented here does not work for atomic energy levels in the general case.