

PHYSICS 102 LAB 10: MEASURING OPTICAL DISPERSION WITH A PRISM
SPECTROMETER
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THEORETICAL DISCUSSION

Dispersion: Light can bend, or *refract*, in a transparent material. The amount of refraction occurring at an interface between two materials depends on the *indices of refraction* for these media— n_1 and n_2 . The index of refraction of a material is defined as the ratio of the speed of light in vacuum, $c = 3.0 \times 10^8$ m/s, to the speed of light v in the material.

$$n = \frac{c}{v}$$

According to Snell's Law, the relationship between the angle that the light makes at the interface between media 1 and 2, θ_1 , and the angle of refraction, θ_2 , is

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where n_1 and n_2 are the indices of refraction of media 1 and 2, respectively.

In general, the index of refraction is not the same for all light wavelengths (colors). This implies that, for a given incident angle, the refraction angle will depend on the wavelength of the incident light. Materials for which this dependence is relatively strong, especially in the visible portion of the light spectrum, are called *dispersive*. The phenomenon whereby the index of refraction of a material measurably depends on the wavelength of the incident electro-magnetic radiation is called *dispersion*.

Prisms and the deviation angle: An optical prism is a chunk of transparent, optically dispersive material which is typically cut in the shape of a triangular wedge. The following discussion is specific to these wedge-shaped prisms; different results may be expected for different prism geometries.

Consider a light ray obliquely incident on side A of a triangular prism such as that shown in figure 1A. This figure shows a typical path for the light ray transiting this material. The angles θ_1 , θ_2 , θ'_1 and θ'_2 are, respectively, the incident angle from medium 1 to medium 2, the refraction angle within medium 2, the incident angle from medium 2 to medium 1, and the refraction angle out into medium 1. They are related according to Snell's law as follows:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_2 \sin \theta'_1 = n_1 \sin \theta'_2$$

The light exits the prism through side B, at an angle θ'_2 with respect to the normal to side B. If we draw a line along the direction of the incident ray through the prism, it will intersect side B at an angle of ω with respect to the normal to that surface. The *deviation angle* δ is defined as the angle between the initial direction of the light ray and its final direction. Inspection of figure 1A shows that this angle is given by

$$\delta = \theta'_2 + \omega$$

It can be shown that

$$\delta = \theta_1 + \theta'_2 - \alpha$$

where α is the angle formed by the leading edge of the prism. If one varies the angle of incidence θ_1 from some arbitrary starting value, either by moving the light source or rotating the prism, the deviation angle δ will become smaller until it reaches a minimum value, after which continued adjustment of θ_1 will cause the deviation angle to increase again. This special, smallest, value of δ is called the *angle of minimum deviation*, which we will denote by the symbol δ_m . The angle of minimum deviation is important because, for this particular angle, we can develop some relatively simple relationships between δ_m and the index of refraction.

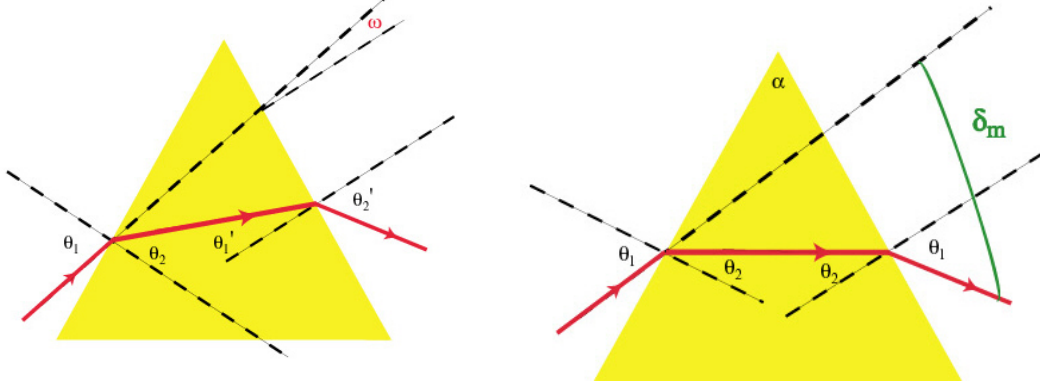


Figure 1A

Figure 1B

FIG. 1: Geometric relations for refracted rays within a prism

The angle of minimum deviation: One can use differential calculus to show that the condition for δ to be a minimum is

$$\theta_1 = \theta_2', \text{ and } \theta_2 = \theta_1'$$

A light ray incident on the prism at an angle θ_1 such that the above condition is satisfied will follow the geometry of figure 1B as it traverses the prism. Further geometric manipulations reveal that when the minimum deviation conditions are satisfied, θ_1 and θ_2 obey rather simple conditions:

$$\theta_2 = \frac{\alpha}{2}$$

and

$$\theta_1 = \theta_2 + \frac{\delta_m}{2} = \frac{\alpha + \delta_m}{2}$$

Taking the index of refraction of air to be $n_1 = 1$, and inserting the above relations for θ_1 and θ_2 into Snell's Law gives a simple equation for the index of refraction of the prism, n_2 , in terms of the angle of minimum deviation:

$$n_2 = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin \left[\frac{1}{2}(\alpha + \delta_m) \right]}{\sin \left(\frac{\alpha}{2} \right)}$$

In a dispersive medium, the index of refraction depends on the wavelength of the incident light λ . By measuring the angle of minimum deviation for different light wavelengths, we can determine the wavelength dependence of the index of refraction by using the equation

$$n(\lambda) = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin \left[\frac{1}{2}(\alpha + \delta_m(\lambda)) \right]}{\sin \left(\frac{\alpha}{2} \right)}$$

The prism spectrometer: The prism spectrometer is shown in figure 2. It consists of a collimator tube, a telescope, a prism, and a circular measuring platter. The collimator tube holds an aperture at one end that limits the light coming from the source to a narrow rectangular slit. A lens at the other end focuses the image of the slit onto the face of the prism. The telescope magnifies the light exiting the prism and focuses it onto the eyepiece. The prism, of course, disperses the incident light into its constituent wavelengths. The measuring platter permits you to read off the angles at which the collimator and the telescope are located.

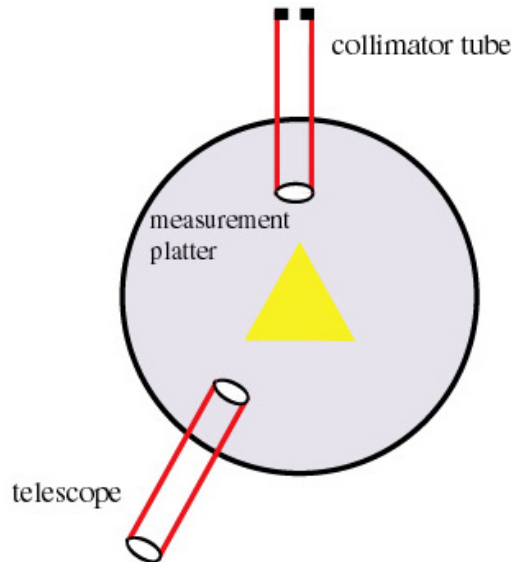


FIG. 2: The prism spectrometer

PROCEDURAL STEPS

In order to extract the wavelength dependence of n for your prism, you will need to determine the angles of minimum deviation for the five different light wavelengths that you found in last week's lab. In principle it is necessary to measure the leading-edge angle α . However, we will just tell you what it is:

$$\alpha = 60^\circ$$

Measuring the angle of minimum deviation δ_m :

1. Remove the prism from the spectrometer and align the telescope so that you can see the illuminated slit in the cross-hairs of the telescope. Adjust the focus on the telescope if necessary.
2. Record the telescope angle and label it θ_0 .
3. Replace the prism so that it is oriented as shown in figure 3. Using the unaided eye (no telescope), locate the image of the atomic spectrum. Narrow the slit as much as you can while still keeping the spectrum visible.
4. Gently rotate the prism back and forth. As you attempt to make the deviation angle smaller, the spectrum should appear to migrate in one direction until a point at which it reverses direction and migrates back to where it came from. Position the prism so that it is roughly at the reversal point.
5. Position the telescope so that you can view the spectrum when the prism is oriented at the reversal point. Make fine adjustments to the prism orientation so as to precisely pin down the reversal point.

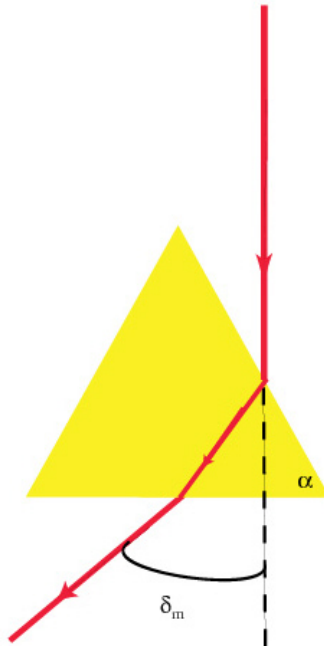


FIG. 3: Orientation of prism and light beam for locating angle of minimum deviation

6. Locate and record the telescope angles $\theta(\lambda)$ for the five different wavelengths that you determined in lab 9.
7. Calculate the angles of minimum deviation $\delta_m(\lambda)$ according to the equation

$$\delta_m(\lambda) = \theta(\lambda) - \theta_0$$

Computing and reporting your results:

1. Tabulate $\delta_m(\lambda)$ and λ for your five wavelengths.
2. Using your measured values of $\delta_m(\lambda)$, calculate the index of refraction as a function of wavelength.
3. Tabulate and plot $n(\lambda)$ vs λ for your five wavelengths.