Two Interacting Populations
Interspecific Competition

Fall 2011

1 Introduction

1.1 Learning Objectives

Through this meeting and the associated readings, you should be able to:

- Describe primary types of interactions between two species;
- Distinguish between *intraspecific* and *interspecific* competition and how to model them;
- Explain how the Lotka–Volterra formulation for *interspecific* competition operates;
- Use state-space graphs and isocline analysis to anticipate possible behavior of dynamic models;
- Identify the assumptions of the Lotka–Volterra competition models;
- Show how intraguild predation changes the Lotka–Volterra competition model; and
- Diagram the species interactions to be modeled.

2 Interaction Types

There are three primary types of relationships (R) one species can have in relation to another: $+$,$-$, and 0. This generates nine possible types of species–species interactions (e.g., $(+,+)$,$(-,+)$,$(-,-)$). We will construct and discuss the $R \times R$ interaction matrix in class. These **interspecific interactions** introduce a number of new processes that we need to consider how to model quantitatively.

3 Lotka-Volterra Interspecific Competition

3.1 Equations

Recall that we modeled density-dependent *intraspecific* competition by adding the logistic function to the exponential growth model as follows:

$$\dot{N}_i = r_i N_i \left(\frac{K_i - N_i}{K_i} \right), \tag{1}$$

Where $\dot{N}_1$ is the rate of population change in species i, $r_i = b_i - d_i$ is the intrinsic growth rate if species i, and K_i is the carrying capacity for species i.

Here we are modifying the logistic equations as suggested by A.J. Lotka and V. Volterra to incorporate the effect of *interspecific* competition between two species (number 1 and 2). This modification is:

$$\frac{dN_1}{dt} = \dot{N}_1 = rN_1 \left(\frac{K_1 - N_1 - f(N_2)}{K_1} \right) \tag{2}$$

$$\frac{dN_2}{dt} = \dot{N}_2 = rN_2 \left(\frac{K_2 - N_2 - f(N_1)}{K_2} \right). \tag{3}$$

We are now working with a system of differential equations that shows how two species are interacting, and we are accounting for the behavior of both species. As formulated above, interspecific competition reduces the growth rate by some function f of the second species.

As is often the case, the interspecific competition function in the equations above $f(N_i)$ can be replaced with many types of functions. However, the simplest and most commonly used function multiplies the competitor density by a constant number as follows:

$$\frac{dN_1}{dt} = \dot{N}_1 = rN_1 \left(\frac{K_1 - N_1 - \alpha N_2}{K_1} \right) \tag{4}$$

$$\frac{dN_2}{dt} = \dot{N}_2 = rN_2 \left(\frac{K_2 - N_2 - \beta N_1}{K_2} \right). \tag{5}$$

Figure 1 shows a Forrester type diagram of the interactions.

In equations (4 and 5), α and β are the *competition coefficients*. As stated in Gotelli (2008) "we can define α as the per capita effects of species 2 on the population growth of species 1, *measured relative to the effects of species 1*" (p. 102).

3.2 Model Assumptions

As we have modified the logistic equations most of the assumptions of the logistic and exponential growth models are also made here. However, there are three additional assumptions we have made:

- Resources are in limited supply;

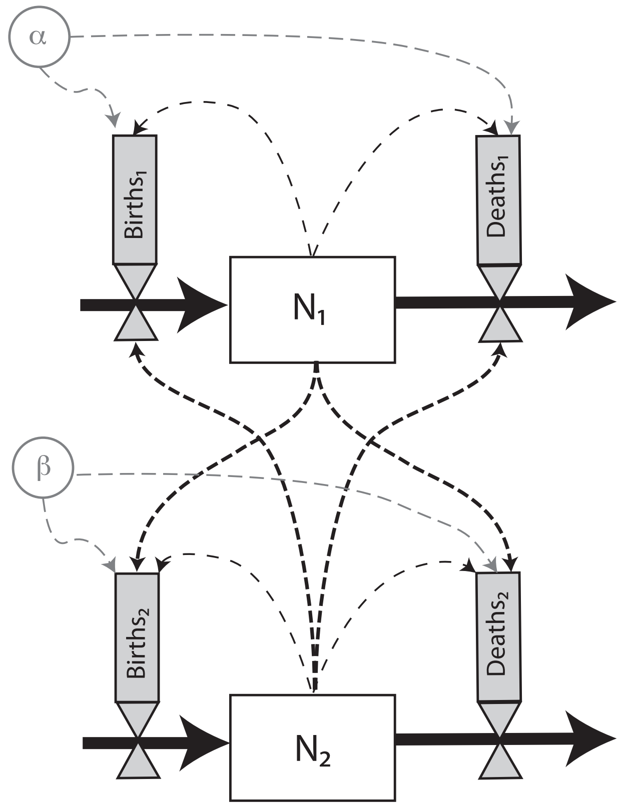

Figure 1: Forrester type diagram for Lotka–Volterra competition.

- Competition coefficients α and β and the carrying capacities K_1 and K_2 are constants; and
- Density dependence is linear.

4 State Space and Isocline Analysis

We can use state space graphs and equilibrium and isocline analysis to anticipate the range of possible behaviors in our system of equations without having to solve them for specific solutions. These are powerful techniques for analyzing the general possible behavior of a model.

4.1 Equilibriums and Isoclines for Lotka-Volterra Competition

As we have done previously, we can find the equilibrium population densities ($\hat{N}$) for each of the Lotka–Volterra competition equations:

$$\hat{N}_1 = K_1 - \alpha N_2 \quad (6)$$
$$\hat{N}_2 = K_2 - \beta N_1 \quad (7)$$

Notice that the equilibriums are no longer points—they are lines in two dimensional space.

4.2 State Space

We normally consider plots of the equations of the state-variable versus time. However, with two interacting state variables, we can also consider the state space, formed by plotting the two state variables against each other (N_1 vs. N_2). A plot of this type is called a *state-space graph*.

Now we can plot the equilibrium lines on the state-space graph. These lines are called "isoclines" because they show a set of abundances of one species for which the other species growth rate is zero. Figure 2 illustrates how each of these isoclines can be plotted on the graph.

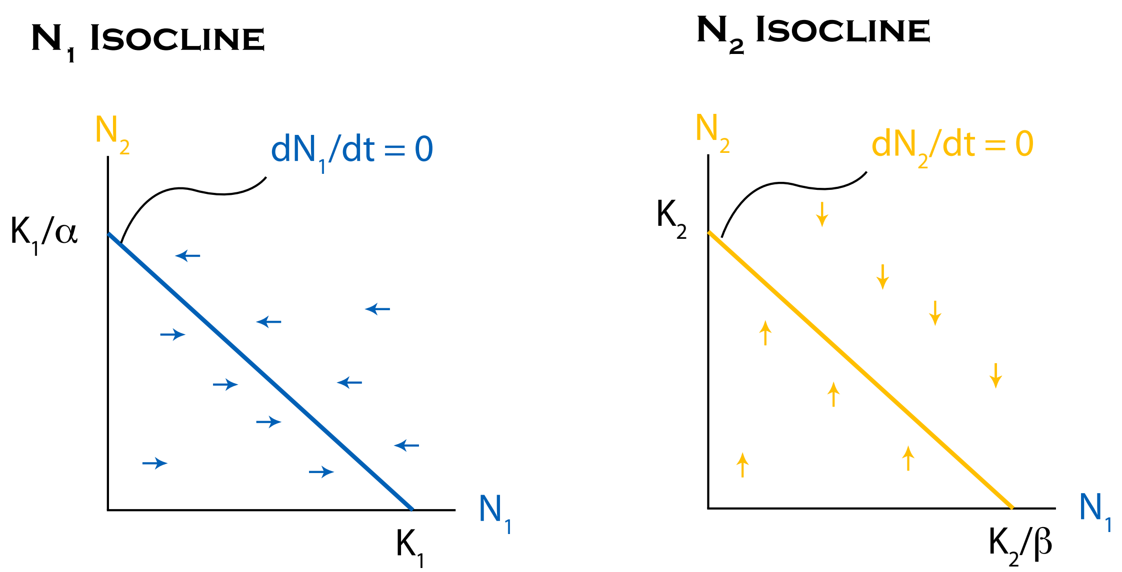

Figure 2: Isoclines

4.3 Graphical Solutions

The isoclines interact to define four possible types of system behavior, depending on the parameter values used. These cases are illustrated in Figure 3.

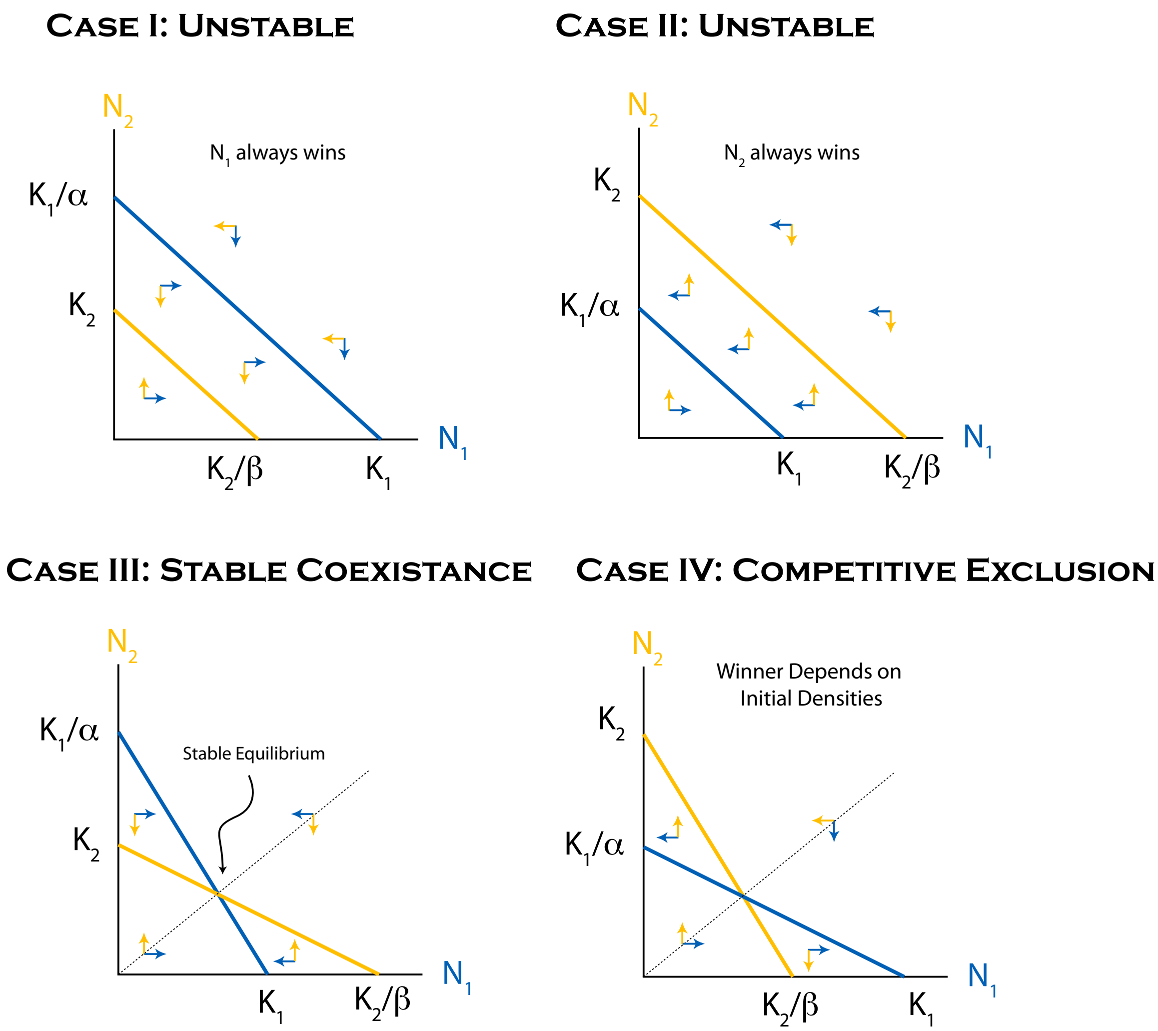

Figure 3: Four possible system configurations for interspecific competition

In three of four cases one species out-competes and excludes the other. According to Kot (2001), this is the basis for *Gause's Principle* or the *Principle of Competitive Exclusion* (p. 203).

Table 5.1 in Gotelli (p. 113) summarizes how to utilize the parameters to determine which case is represented in a particular model.

5 Model Variation: Intraguild Predation

5.1 Diagram

See Figure 4.

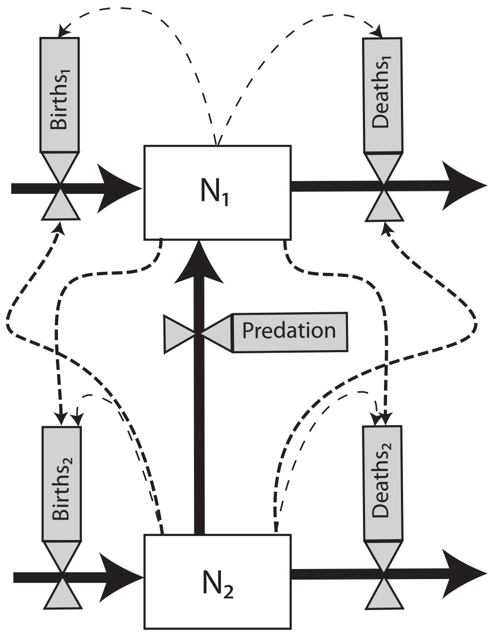

Figure 4: Intraguild Predation

5.2 Equations

$$\frac{dN_1}{dt} = \dot{N_1} = rN_1\left(\frac{K_1 - N_1 - \alpha N_2}{K_1}\right) + \gamma N_1 N_2 \tag{8}$$

$$\frac{dN_2}{dt} = \dot{N_2} = rN_2\left(\frac{K_2 - N_2 - \beta N_1}{K_2}\right) - \delta N_1 N_2 \tag{9}$$

In these equations γ and δ are interaction coefficients.

5.3 Isoclines

See Figure 5.9 in Gotelli on page 117.

References

Gotelli, N.J. 2008. A Primer of Ecology (fourth edition). Sinauer Associates, Inc., Sunderland, MA.

Kot, M. 2001. Elements of Mathematical Ecology. Cambridge University Press, Cambridge UK.