Laboratory 5: Discrete Time Population Models and Chaotic Dynamics

Biol 535

Introduction

The Ricker model is a discrete-time analogue of the continuous time logistic model that was first formulated by Ricker (1954) to model fisheries stocks. The update rule is

$$N_{t+1} = aN_t e^{-bN_t} \tag{1}$$

where N_t is the number of mature individuals at time t in years. Gurney and Nisbit (1998) describe their application of the model to a fishery as follows:

> "We work with a time increment (Δt) of 1 year, and denote the stock of mature individuals at the census date in year t by X_t. Juveniles mature the year after their birth. Adult fish spawn once before dying and produce a maximum of [a] viable recruits to the following year's stock. Due to cannibalism on eggs by adults, the juvenile survivorship in a year when there are X_t adults is e^{-bX_t}, where b is a parameter related to the intensity of cannibalism." (p. 27)

Task 5.1: Parameter Sensitivity Your first task is to write an R script to encode the Ricker model described above using a for loop. To begin, let $N_0 = 100$, $a = 0.8$, and $b = 0.001$. Then, investigate how the model changes when you change parameter b. Reset b to 0.001 and explore how the behavior changes as you increase a to about 20. What do you observe? Describe the behavior changes you see and document your observations with plots. What do you surmise is happening in this deterministic model (hint: see reading from Otto and Day)? How does this compare to your solutions for the continuous time logistic equation?

Task 5.2: Initial population size Reset your model parameters to $a = 18$ and $b = 0.001$ and now lets explore the effect of changing the initial population size. To start, plot the population dynamics predicted when $N_0 = 100$ and when $N_0 = 101$ on the same graph. Use a time span from 0 to 30 years. What do you observe? Try a couple of additional values.

Exploring the "sensitivity" of the model solution to changes in parameters and initial conditions as you are doing in this laboratory is a simple form of sensitivity analysis, and is a powerful technique to learn about the functions or models you build or use.

Challenge Problem If you have time and energy after completing Task 5.1 and 5.2, I present the following as a puzzle to challenge your analytical and programming skills.

Bifurcation diagrams like Figure 4.1.2 in the Otto and Day reading (p. 118) are often used to describe and summarize chaotic dynamics. See if you can create a bifurcation diagram for the Ricker model.

References

Gurney, W. S. C., and R. M. Nisbet. 1998. Ecological dynamics. Oxford University Press, New York.

Ricker, W. E. 1954. Stock and recruitment. Journal Fisheries Research Board of Canada 11:624-651.